Parallelized Algorithm for the Computation of N-gram Specific Absorption Rate (SAR) on a Graphics Processor

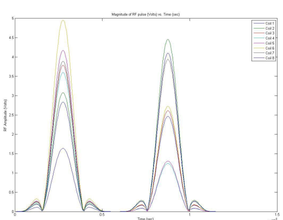
L. KINI[1], M. GEBHARDT[2], K. MAKHOUL[3], L. L. WALD[3,4], AND E. ADALSTEINSSON[1,4]

[1]EECS, MASSACHUSETTS INSTITUTE OF TECHNOLOGY, CAMBRIDGE, MA, UNITED STATES, [2]SIEMENS MEDICAL SOLUTIONS, ERLANGEN, GERMANY, [3]MGH, HARVARD MEDICAL SCHOOL, A. A. MARTINOS CENTER FOR BIOMEDICAL IMAGING, CHARLESTOWN, MA, UNITED STATES, [4]HARVARD-MIT DIVISION OF HEALTH SCIENCES AND TECHNOLOGY, MIT, CAMBRIDGE, MA, UNITED STATES

INTRODUCTION. Estimation and management of local Specific Absorption Rate (SAR) is a dominant challenge in parallel transmission (pTx) for high field MRI. Real-time estimates of local and global SAR over high-resolution volumes for a given patient, RF pulse and coil configuration are highly desirable but are computationally challenging with conventional processors. Low-resolution human models are computationally more feasible but sacrifice accuracy. Brute-force methods of calculating local SAR values, such as by assuming fixed cube regions as approximations to *N*-gram regions, have been shown to be less accurate than algorithms that incorporate tissue density for N-gram volume estimates[1]. A recent method to calculate the maximum *N*-gram SAR proposed an algorithm to efficiently calculate *N*-gram regions using region growth [1]. This algorithm, though both faster and more accurate than the brute-force method, can still be improved in memory and runtime, especially when applied to higher-resolution models [1]. We propose and demonstrate improvements in the region-growth algorithm by exploiting its inherent parallel computation structure implement it on a commercial graphical card (with GPUs) for faster runtime.

METHODS Point-wise SAR. We define "point SAR" as the time-average SAR at every voxel $\mathbf{r} = [x, y, z]$ due to the parallel transmission of RF pulses $a_1(t), \cdots, a_P(t)$ on a *P*-channel pTx system. We use a sampling period of Δt, resulting in a maximum of N_t RF time samples, with duration $L = N_t\,\Delta t$. We assume a known $\mathbf{E}_p(\mathbf{r}) = [\,E_{p,x}(\mathbf{r}),\ E_{p,y}(\mathbf{r}),\ E_{p,z}(\mathbf{r})]$, the 3-dimensional $(n_x \times n_y \times n_z)$ electric field at $\mathbf{r}$ due to a unit amplitude pulse in coil *p*. We calculate point SAR at $\mathbf{r}$ as a superposition of the square of the magnitude of the electric field produced by each RF pulse, weighted by conductivity and density, and then time average this quantity over the duration of the pulse [2]. **N-gram SAR**. Local *N*-gram SAR at $\mathbf{r}$ is calculated by finding *N*-gram cubes around $\mathbf{r}$ and then averaging the point SAR($\mathbf{r}$) for all voxels in that *N*-gram cube. The *N*-gram cube is estimated by a fast region-growth algorithm [1] implemented on a graphical processor. We store in GPU memory the subset of offsets such that the total mass of all the offset voxels is approximately *N* grams. In addition, a cross-correlation power matrix of the RF pulses (see Figure 1, left column) is precalculated to remove dependency of runtime on number of time samples, N_t[4]. The theoretical average runtime of this algorithm is $O(M \log M + N_t P^2 + P^3)$ and GPU memory cost is $O(n_x \times n_y \times n_z + M)$. We assume that the number of tissue types in the model is negligible in terms of memory, which is a good assumption for our current classification (28 tissue types).

$$
\begin{aligned}
SAR(\mathbf{r}) &= \frac{\sigma(\mathbf{r})}{2\rho(\mathbf{r})}\frac{1}{L}\int_0^L \|\mathbf{E}(\mathbf{r},t)\|_2^2\,dt \\
&\approx \frac{\sigma(\mathbf{r})}{2\rho(\mathbf{r})}\frac{1}{N_t}\sum_{n=0}^{N_t-1} \|\mathbf{E}(\mathbf{r},n\Delta t)\|_2^2 \\
&= \frac{\sigma(\mathbf{r})}{2\rho(\mathbf{r})}\frac{1}{N_t}\sum_{n=0}^{N_t-1} \|\sum_{p=1}^{P} a_p(n\Delta t)\mathbf{E}_\mathbf{p}(\mathbf{r})\|_2^2 \\
&= \frac{\sigma(\mathbf{r})}{2\rho(\mathbf{r})}\frac{1}{N_t} trace\left\{\mathbf{E}_p^H(\mathbf{r})\left[\sum_{n=0}^{N_t-1}\mathbf{a}^H(n\Delta t)\mathbf{a}(n\Delta t)\right]\mathbf{E}_p(\mathbf{r})\right\} \\
&= \frac{\sigma(\mathbf{r})}{2\rho(\mathbf{r})}\frac{1}{N_t} trace\left\{\mathbf{E}_p^H(\mathbf{r}) \times \mathbf{\Gamma} \times \mathbf{E}_p(\mathbf{r})\right\}
\end{aligned}
$$



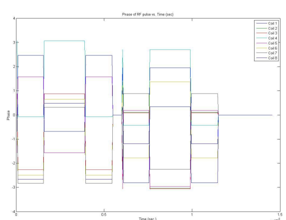


Figure 1a. RF Pulse magnitude for each coil in Volts over 291 time samples, Δt = 5 us. Figure 1b. RF Pulse phase for each coil.

Results. SAR computation runtime was measured for three methods for *N*=10-gram SAR calculation of 8-channel loop-array for the HUGO head model at 3mm resolution[3]. Table 1 shows run-time of brute-force region growth (accurate but intractable), fast region growth [1], and the GPU-assisted region growth ultrafast SAR computation for the N_t = 291 RF pulse for *P*=8 coils shown in Fig 1 using E fields obtained from FDTD simulations. The E fields were interpolated to simulate higher resolution models (from 3mm resolution with $n_x = n_y = 256$ $n_z = 128$ which takes up 33 MB card memory to 0.75 mm resolution with $n_x = n_y = 1024$ $n_z = 512$ which takes up 2 GB card memory). Table 2 shows run-time of the same three algorithms on RF pulses of different number of time samples, from N_t = 291 to N_t = 900 on the same E fields at 3 mm resolution ($n_x = n_y = 256$ and $n_z = 128$). **Conclusion.** A fast accurate method of calculating N-gram SAR has been demonstrated using GPUs with up to 4x acceleration. Runtime does not depend linearly on number of RF time samples N_t but memory depends on **E** resolution.

Disclaimer

The concepts and information presented in this paper are based on research and are not commercially available.

RESOLUTION (nx,ny,nz) ⇒ / ALGORITHM ⇓	(256, 256, 128)	(512, 512, 256)	(1024, 1024, 512)
Brute-Force Region Growth	10 min.	~ 72 min.	~ 7 hrs.
CPU Fast Region Growth [1]	5 min.	24 min.	~2 hrs.
GPU-assisted ultrafast region growth	2 min.	6 min.	18 min.

Number of RF Time Samples in each coil ⇒ / ALGORITHM ⇓	N_t=300	N_t=600	N_t=900
Brute-Force Region Growth	10 min.	21 min.	31 min.
CPU Fast Region Growth [1]	5 min.	11 min.	15 min.
GPU-assisted ultrafast region growth	2 min.	2 min.	2.5 min.

Table 1. Runtime for 3 algorithms for 10-g SAR calc. on different E field resolutions on same RF pulse
Table 2. Runtime for 10-g SAR calc. on different number of RF time samples on same E field.

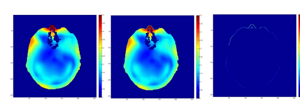


Fig 1. 10-g SAR maps using Brute-force region growth and proposed GPU algorithm with difference plot.

[1] A. C. Zelinski, et al. Fast, Accurate Calculation of Maximum Local N-Gram SAR. (ISMRM) 2008. [2] Zelinski, A. C. Improvements in MRI Excitation Pulse Design. MIT PhD thesis. [3] C. Gabriel. "Compilation of the dielectric properties of body tissues at RF and microwave frequencies," Brooks AFB, TX, 1996. [4] Matthias G. submitted. ISMRM 2010

Acknowledgments. NIH R01EB006847 & NIH R01EB007942, Siemens Medical Solutions.