A Novel MRI Framework for the Quantification of Any Moment of Arbitrary Velocity Distributions

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Introduction: Characterization and quantification of blood flow dynamics is of pivotal importance to the understanding and detection of cardiovascular disease. MRI can determine several important flow dynamics parameters but might not yet have reached its full potential. In practice, each voxel in an object contains a distribution of spin velocities, s(v). Distributions can be characterized by their moments (table 1). Under the assumption that $s(v)$ is symmetric about its mean, the data obtained from a phase-contrast (PC) MRI measurement permit an estimation of the first raw moment of $s(v)$, i.e. its mean velocity, V_m [1,2]. In this work, we present a generalized framework for the quantification of *any* moment of an *arbitrary* spin velocity distribution $s(v)$ and describe how this framework relate to existing MRI methods for the assessment of fluid flow.

Theory: For an arbitrary spin velocity distribution $s(v)$, the n^{th} raw moment is given by $\mu'_n = {}_{-\infty}\int^{\infty} v^n \cdot s(v)dv$ and the n^{th} central moment is given by $\mu_n = {}_{-\infty}\int^{\infty} (v - \mu'_1)^n \cdot s(v)dv$ (table 1). Including motion terms up to the first order, the complex-valued MRI signal of a voxel can be written as the Fourier transform

$$S(k_v) = Ce^{i\varphi_{\text{add}}} \int_V s(v) e^{-ik_v v} dv \quad \text{[Eq. 1]},$$

where C is a real-valued constant describing spin density, relaxation effects, etc, φ_{add} is a phase-shift caused by for example field inhomogeneities, and k_v is proportional to the first moment of the gradient waveform (VENC = π / k_v). Moment in the function domain (velocity-space) corresponds to derivation in the transform domain (k_v-space) at $k_v = 0$. Thus, to obtain the n^{th} moment of a voxel without making assumptions about the distribution s(v), a measurement of the n^{th} derivative of S(k_v) at $k_v = 0$ needs to be determined.

Materials and Methods: To demonstrate the application of the moment framework, a non-Gaussian asymmetric intravoxel velocity distribution $s(v)$ (Fig. 1a) was extracted from computational fluid-dynamics data describing post-stenotic flow [3]. The corresponding MRI signal $S(k_v)$ was generated by computing Eq. 1 for a range of k_v-values (Fig. 1 b and c). By utilizing finite differentiation, the n^{th} derivative of $S(k_v)$ (n^{th} moment of s(v)) can be approximated from a minimum of n+1 measurements of $S(k_v)$. The accuracy of the finite differentiation approximation, with and without noise, was investigated as a function of the spacing, Δk_v for the estimation of mean velocity and variance.

It can be shown that the first derivative of $S(k_v)$ approaches the derivative of arg(S) when $k_v \rightarrow 0$. Assuming that s(v) is symmetric about its mean implies that arg($S(k_v)$) is linear on the interval $|k_v| < \pi/V_m$. In this way, the PC-MRI phase-difference method can be used to estimate the mean velocity V_m. Similarly, by modeling a specific distribution of $s(v)$ and fitting the model parameters to measurements with larger Δk_v, PC-MRI intravoxel velocity standard deviation (IVSD) mapping [4] can by used to estimate the standard deviation σ.

Results: A non-Gaussian asymmetric spin velocity distribution s(v) along with the modulus and argument of the corresponding MRI signal $S(k_v)$ is shown in Fig. 1a-c. A small Δk_v (good approximation of the derivative) provides accurate estimates of V_m (Fig. 2a) and σ (Fig. 3a). In presence of noise, measurements of S(k_v) at $k_v \sim 0$ become indistinguishable, resulting in unreliable estimates (Fig. 2b and 3b). Employing the PC-MRI phase-difference and IVSD methods, more accurate estimations of V_m (Fig. 2c) and σ (Fig. 3c), respectively, can be obtained with greater Δk_v.

Conclusion: We have presented a generalized framework for the quantification of any moment of arbitrary intravoxel spin velocity distributions. The relationship between this framework and the existing PC-MRI phase-difference and IVSD methods was described. The presented moment framework may assist in improving the understanding of existing MRI methods for the quantification of flow and motion and be a fertile ground for the development of new methods.

Table 1. Examples of moments of a distribution

Moment	Relation to common quantities
First raw moment, μ'_1	Mean, $V_m = \mu'_1$
Second central moment, μ_2	Standard deviation, σ = sqrt(μ_2)
Third central moment*, μ_3	Skewness, $\gamma_1 = \mu_3/\sigma^3$
Fourth central moment*, μ_4	Kurtosis, $\gamma_2 = \mu_4/\sigma^4$

* Also known as the 3rd and 4th standardized moments, respectively.

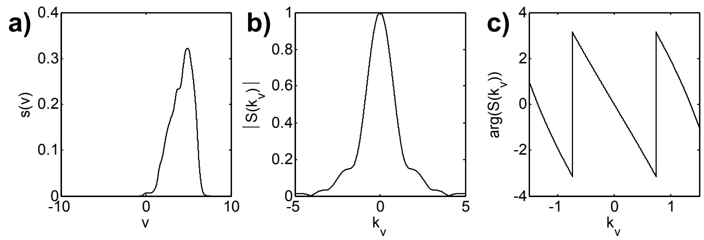

Fig. 1. a) A non-Gaussian asymmetric spin velocity distribution s(v) along with b) the modulus and c) the argument of the corresponding MRI signal S(k_v).

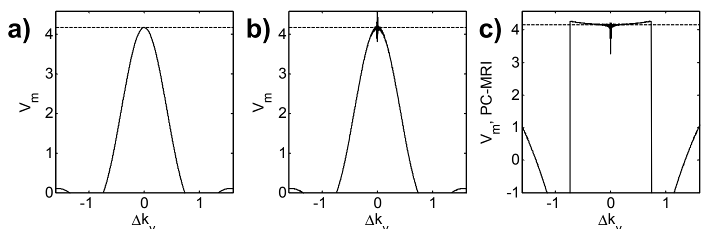

Fig. 2. The mean of s(v), V_m, as obtained by finite difference approximations of the first derivative of S(k_v) at k_v = 0 in a) absence of noise and b) presence of noise, and c) as obtained by the PC-MRI phase-difference method. The dashed lines indicate the true V_m. In c) the deviations of the PC-MRI estimates from the true V_m are due to the assumption that s(v) is a symmetric distribution. At $|\Delta k_v| > 0.7$, velocity aliasing occurs.

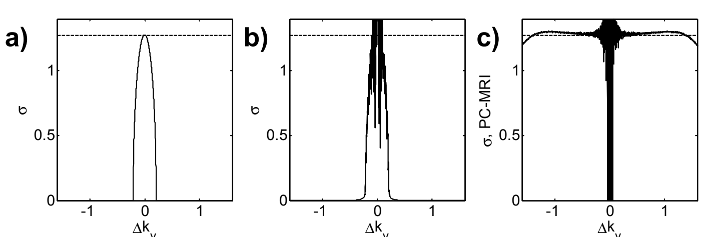

Fig. 3. The standard deviation of s(v), σ, as obtained by finite difference approximations of the second derivative of S(k_v) at k_v = 0 in a) absence of noise and b) presence of noise, and c) as obtained by the PC-MRI IVSD method. The dashed lines indicate the true σ. In c) the deviations of the PC-MRI estimates from the true σ are due to the assumption that s(v) is a Gaussian distribution

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