

Probabilistic Flow Connectivity Mapping

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ABSTRACT Phase-Contrast (PC) MRI utilizes phase shifts resulting from moving spins to quantify tissue motion and blood flow. In cardio-vascular PC MRI applications, visualization techniques such as vector glyphs, streamlines, and particle traces are employed for depicting the flow situation. Whereas these techniques indeed provide useful information, they ignore the noise in the PC-MRI measurements and may even give a false sense of precision. With the aim of visualizing uncertainty in PC MRI, this work contributes a probabilistic flow connectivity mapping method in which possible flow traces are sampled using a sequential Monte Carlo approach.

NOISE IN PC MRI MEASUREMENTS In a noise free PC MRI case, the flow velocities in one direction, say x , are encoded in the phase of a complex image $S_x = Ae^{i(\varphi - \frac{\pi}{v_{enc}}v_x)}$, where v_x is the true velocity in every voxel and v_{enc} is a sequence parameter setting the maximal measurable velocity. φ denotes a constant velocity encoding term as well as a static but uncontrollable spatially varying phase shift. To cancel φ , a second image $S_0 = Ae^{i\varphi}$ is acquired, whereby the velocity can be found as $v_x = \frac{v_{enc}}{\pi} [\arg(S_0) - \arg(S_x)]$ (Eq. 1). In practice, independent Gaussian noise $n \in N(0, \sigma^2)$ contaminates both the real and imaginary parts of the measurement so that $S_0 = Ae^{i\varphi} + n_0^r + in_0^i$ and $S_x = Ae^{i(\varphi - \frac{\pi}{v_{enc}}v_x)} + n_x^r + in_x^i$. In [1], it is shown that, in high SNR areas ($A \gg \sigma$) such as the blood pool, the distribution of $\arg(S_0) \in N(\varphi, \frac{\sigma^2}{A^2})$ and similarly for $\arg(S_x)$. Hence, the observed velocity \tilde{v}_x , calculated according to Eq. 1, may be seen as drawn from the distribution $\tilde{v}_x \in N(v_x, \frac{v_{enc}^2 2\sigma^2}{\pi^2 A^2})$. In a 3D PC-MRI flow measurement, S_y and S_z images are acquired as well, and the velocities \tilde{v}_y and \tilde{v}_z are estimated analogously to Eq. 1. The joint distribution of the measured 3D vector $\tilde{\mathbf{v}} = [\tilde{v}_x, \tilde{v}_y, \tilde{v}_z]^T$ becomes multivariate Gaussian $\tilde{\mathbf{v}} \in N(\mathbf{v}, \mathbf{C})$. In [2], the univariate distribution of the flow magnitude $\|\tilde{\mathbf{v}}\|$ is derived for a diagonal isotropic covariance matrix. However, in the current case, the full multivariate distribution is required and the covariance matrix has a more complicated structure $\mathbf{C} = \frac{v_{enc}^2 \sigma^2}{\pi^2 A^2} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ because the image S_0 is involved in the calculation of all three velocity components. To fully specify \mathbf{C} , estimates of the noise variance σ^2 and image magnitude A are required. Manual and automatic methods for this estimation using a magnitude image, in the PC MRI case an average of the magnitudes of S_0, S_x, S_y , and S_z images, are described in [3]. In this work, σ^2 and A are estimated from manually delineated regions in the air and blood pool respectively.

FLOW CONNECTIVITY MAPPING A conventional streamline or particle trace algorithm uses $\tilde{\mathbf{v}}$ as estimate of the true velocity and draws a flow path of a virtual particle that may be seen as the most likely in a maximum likelihood sense. In this work, the uncertainty associated with such a path is addressed. To this end, a sequential Monte Carlo approach is employed, in which trajectories are generated by iteratively sampling flow vectors from the multivariate Gaussian distribution described above. Using $\tilde{\mathbf{v}}$ as estimate of the true velocity, the procedure is similar to a streamline or particle trace algorithm, but with the addition of a random perturbation vector drawn from $N(0, \mathbf{C})$ in every tracking step. The generated trajectories are referred to as probabilistic streamlines or particle traces respectively. Each generated trajectory initiated at a point \mathbf{x} may be seen as a sample from a global probability distribution of all possible paths a virtual particle may take from \mathbf{x} . To visualize this distribution, a large number of trajectories (>1000) initiated at \mathbf{x} are generated. From these trajectories, a flow connectivity map can be calculated by counting the number of trajectories that pass through each voxel and normalizing with the total number of trajectories. For time-resolved 4D PC MRI data, the connectivity map is also 4D and shows the probability of finding the virtual particle in a particular voxel and time interval.

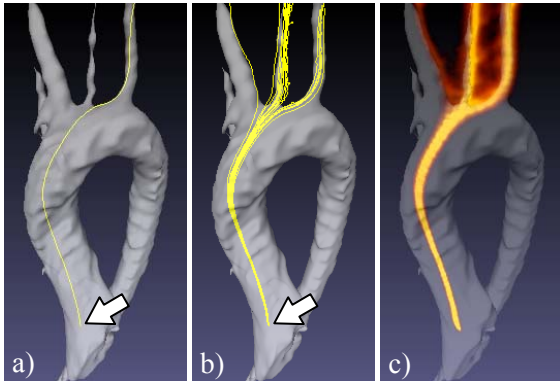


Fig 1. a) A regular streamline initiated at the arrow. b) 50 probabilistic streamlines from the same point. c) A flow connectivity map based on 4000 probabilistic streamlines.

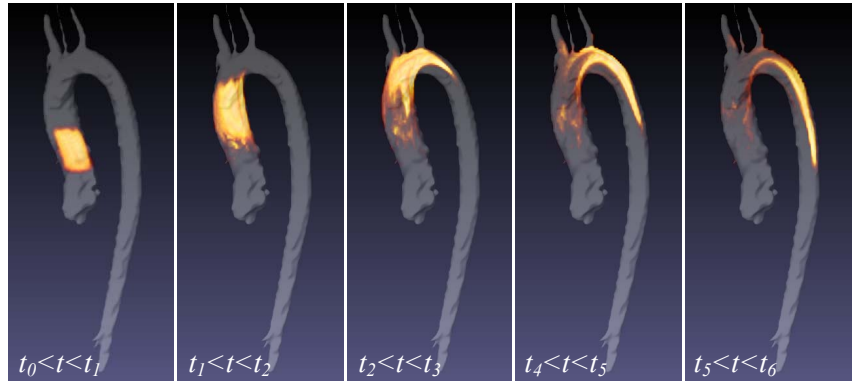


Fig 2. A 4D flow connectivity map based on probabilistic particle traces using an entire cross-section as seed region. The colors may be interpreted as the likelihood of finding a virtual particle in a voxel and time interval. The length of the time intervals is 48.8 ms.

RESULTS AND DISCUSSION The flow connectivity mapping method is demonstrated using a 4D PC MRI data set of the aorta acquired with the following parameters: TE/TR 3.67/6.1 ms, flip angle 15°, $v_{enc} = 150$ cm/s, spatiotemporal resolution $1.7 \times 1.7 \times 3.5$ mm³ and 48.8 ms. The noise variance was estimated to $\sigma^2 \approx 25$ and the image magnitude in the blood pool to $A \approx 55$. Fig. 1a shows a regular streamline that traces the velocity field from a seed point in the ascending aorta into the left subclavian artery. For comparison, Fig. 1b shows 50 probabilistic streamlines initiated at the same point, illustrating different paths that may be taken when noise is accounted for. The results clearly demonstrate that the noise uncertainty can even lead to traces reaching other supra-aortic vessels, which was not indicated by the regular streamline. The flow connectivity map in Fig. 1c is based on 4000 probabilistic streamlines and shows the detailed flow distribution into the brachiocephalic, carotid, and subclavian arteries. Fig. 2 shows a 4D connectivity map based on probabilistic particle traces seeded in a cross-section of the ascending aorta. A first application of the proposed probabilistic flow connectivity mapping method is as a novel visualization technique that addresses the uncertainty in the PC MRI measurements. New probabilistic measures for quantifying possible flow trajectories are also envisaged, e.g., to assign a likelihood to a flow pathway connecting two vascular regions of interest to evaluate mixing of blood, or to quantify embolization probabilities in stroke and infarction.

REFERENCES [1] H. Gudbjartsson et al. *Magn. Reson. Med.* (1995), 34(6):910-914. [2] A.H. Andersen et al. *Med. Phys* (1996), 23(6):857-869. [3] J. Sijbers. Ph.D Thesis (1998).