Fast and Accurate Large-Tip-Angle RF Pulse Design for Parallel Excitation Using a Perturbation Analysis of the Bloch Equation

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Introduction: Designing RF pulses in parallel excitation (PTX) using the linear small-tip-angle approximation [1-3] produces distorted excitation patterns at large tip angles due to the nonlinear nature of the Bloch equation. Grissom *et al* [4] proposed the 'Additive Angle' (AA) method for large-tip-angle RF pulse as a means to improve upon the small-tip-angle approximation (STA). This iterative method produces an improved profile by applying a STA to the error in the magnetization profile. In this work, we demonstrate that through a perturbation analysis to the STA, it is possible to develop an exact formula that relates the error in the magnetization profile to the required B_1 field. This formula is then used as the basis for a *fast and accurate* method for large-tip-angle RF pulse design.

Method: The proposed method is analogous in concept to the 'Additive Angle' method [4], however, it is derived from a perturbation analysis to the Bloch equation. Given a desired transverse excitation pattern, $M_D(x)$, we design initial RF pulses, $B^P_1(t)$, via the small-tip-angle parallel pulse design method of [3]. If we denote by $M^P(x)$ the transverse magnetization at spatial position (x) created by $B^P_1(t)$, then by inverting the Bloch equation,

$$\Delta M(x_i)=i\gamma\sum_{r=1}^{R}S_r(x_i)\int_0^T M_z^p(x_i,t)\Delta\vec{b}_r(t)e^{i2\pi\vec{k}(t)\cdot\vec{x}}dt + i\gamma\sum_{r=1}^{R}S_r(x_i)\int_0^T \Delta M_z^p(x_i,t)\vec{b}_r^p(t)e^{i2\pi\vec{k}(t)\cdot\vec{x}}dt + i\gamma\sum_{r=1}^{R}S_r(x_i)\int_0^T \Delta M_z^p(x_i,t)\Delta\vec{b}_r(t)e^{i2\pi\vec{k}(t)\cdot\vec{x}}dt \qquad (1)$$

where $\Delta M=M_D-M_P$ denotes the difference between the desired and excited transverse magnetization, $B_1=B_1^P+\Delta B_1$ is the updated RF pulse and M_z^P represents the longitudinal magnetization created by B_1^P. This formula improves upon the 'Additive Angle' method since it does not perform and STA on ΔM. In fact, by making a STA for ΔM and ignoring the last two terms, the formula of Ref. [4] is obtained. Using Eq. 1, we can iteratively design pulses by minimizing the cost function,

$$\Delta\vec{B}_1\{\Delta b_1,...,\Delta b_r\}=\underset{\Delta b_r}{argmin}\left\{\|\sum_{r=1}^{R}S_r(M_z^p+\Delta M_z^p)A\Delta b_r-\Delta M^{new}\|_W^2+\beta\sum_{r=1}^{R}\|b_r+\Delta b_r\|^2\right\} \qquad (2)$$

$$\Delta M^{new}=\Delta M-\sum_{r=1}^{R}S_r\Delta M_z^p A b_r \qquad (3)$$

where $\Delta M_z^p = M_z^{p,j+1}-M_z^{p,j}$ (*j* is the iteration number), and other parameters are as in Ref.[3]. In the proposed method, Eq. 2 is used to calculate RF pulse corrections via the CG algorithm, and these updated RF pulses are then used to drive an exact Bloch equation simulator. The results from the Bloch simulator are then used to calculate ΔM, which is, in turns, used to calculate further RF pulse updates. This process is repeated until the excitation error reaches a minimum.

Methods: We designed 90° excitation using spiral and echo-planar (EP) trajectories in MATLAB R2009a (Mathworks, Natick, MA, USA). One of the desired excitation patterns (Fig.1) was a smoothed rectangle with FOV 20cm and resolution 0.3125cm, the other (Fig. 5) was our institution's logo. Transmit Sensitivity maps were obtained on a 7T SIEMENS scanner equipped with an 8-channel PTX extension. Comparisons with all methods were examined with regularization parameters (β) of 10 and speedup factors (R) of 6 except in Fig 4, where R was varied.

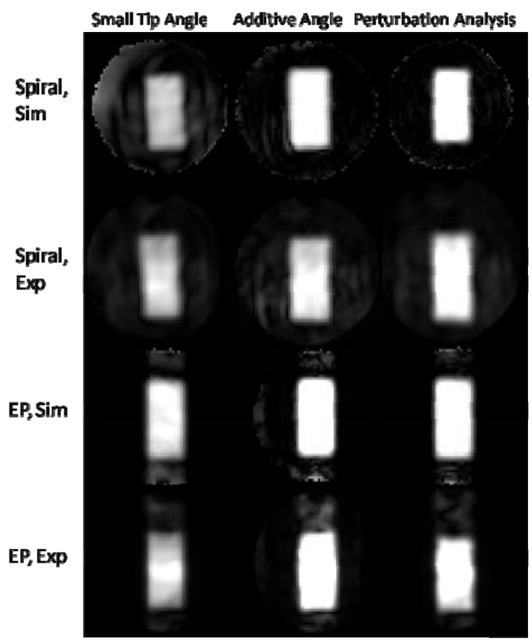

Fig.1 Excitation Patterns using different methods

Results and Discussion: Figure 1 shows a comparison of the transverse magnetization patterns excited by different methods. In the spiral case, the pattern excited by STA method is tilted. This tilt is eliminated when the AA or PTA method are used, though the AA method produces larger excitation errors. A more in-depth look at the results is provided by looking at the profiles from the midline of these excitation patterns (Fig. 2). In the EP case, the excitation pattern of the STA method is not homogeneous. The AA method excites a homogeneous pattern by compromising the accuracy outside. Further improvement is then obtained using the PTA. Notably, the method proposed here improves the results (RMSE=0.0156 vs 0.0289) while at the same time converging faster (Fig. 3). Figure 4 presents a comparison of excitation accuracy for these latter two methods over a range of speedup factors. For both spiral and EP trajectories, the PTA method achieves lower excitation error. Figure 5 shows an experimental comparison on the target magnetization profile of our institution's logo when excited by the STA and PTA methods using a spiral trajectory. Note that, in general, simulations results are of better quality for the spiral designs than for the EP designs while the converse is true for all experimental results.

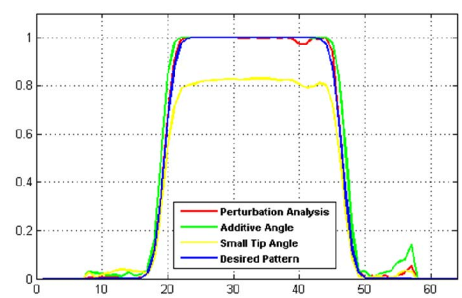

Fig 2. Midline profiles of excitation patterns in Fig 1

Conclusion: We have introduced a fast iterative method for large-tip-angle RF pulse design in PTX and demonstrated its ability to decrease excitation errors for both spiral and EP trajectories. The proposed method relies on an iterative scheme for the solution of an exact inversion of the Bloch equation. Consequently, it is not limited in its choice of k-space trajectories and it converges very fast.

References: [1] U. Katscher *et al.*, MRM 49:144-150, 2003. [2] Y. Zhu, *et al.*, MRM 51:775-784, 2004. [3] W. Grissom *et al.*, MRM 56:620-629, 2006. [4] W. Grissom *et al.*, MRM 59:779-787, 2008. [5] D. Xu *et al.*,MRM 58:326-334.2007

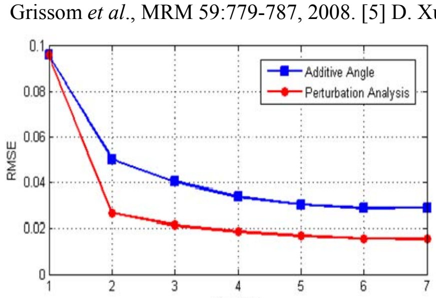

Fig.3 Comparison of convergence for two methods

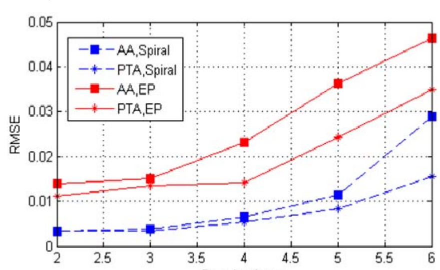

Fig.4 Excitation error of two methods over a range of speedup factors

Figure 5 experiments using Small Tip Angle (left) and Perturbation analysis (right)