Metric Selection and Variability Maps for Diffusion Tensor Data

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Introduction

The selection of a distance function (metric) between tensors is the first step to developing a statistical framework to characterize their variability, permitting group comparisons and statistical inferences based on the entire tensor. Such a tensor-variate statistical framework would subsume univariate statistical distributions for scalar tensor-derived quantities, such as the FA or ADC; those can only account for a fraction of the variability. A tensor-variate statistical framework was proposed in [1], where a Euclidean distance was used to estimate a 6x6 covariance matrix for diffusion tensor data. Recently, a new approach to measure distances between tensors was proposed, which assumes they reside on a Riemannian manifold that requires an affine-invariant metric [2-4]. The Log-Euclidean distance was then shown to approximate the affine-invariant metric without the high computational overhead [5]. Since the Log-Euclidean metric resides on a Euclidean manifold, calculation of its covariance matrix is possible [6]. Here we compare variability maps obtained for the Euclidean and Log-Euclidean metrics using synthetic and real data. We show that the Log-Euclidean distance **does not adequately model the effect of Rician** noise in diffusion weighted imaging data. The Log-Euclidean variability maps are over-estimated in white matter, and underestimated in CSF. We suggest that the Euclidean metric provides a more democratic measure of tensor variability appropriate for tensor based group statistics.

Methods

High resolution DTI was acquired on a 16-channel 3T scanner (GE-Signa) using a PGSE-EPI sequence with TR/TE of 8500/80.9 ms, matrix size of 128x128 and 1.4mm^3 voxel size. In order to improve SNR measurements were repeated 8 times. All results reported here relate to a single slice from this dataset. All DWIs were corrected for head motion using rigid body transformations (SPM2, UCL) and the gradient orientations were compensated for solid body rotation. A tensor map was calculated for each repetition, and a mean tensor map was calculated from all the DWIs. Tensor estimation was done using a Log-Euclidean fit [7] that assures all tensors are positive-definite. We preformed Monte-Carlo (MC) simulations to synthesize 100 noisy replicates of the mean tensor map. The 6x6 Euclidean (C_{Euc}) and Log-Euclidean ($C_{Log\text{-}Euc}$) covariance matrices were estimated using

$$C_{Euc} = 1/(N-1)\sum_{m=1}^{N}\left(D_m - \overline{D}\right)\left(D_m - \overline{D}\right)^T \text{ [1] and } C_{Log-Euc} = 1/(N-1)\sum_{m=1}^{N}\left(\log(D_m) - \log(\overline{D})\right)\left(\log(D_m) - \log(\overline{D})\right)^T \text{ [6].}$$

The Trace of the covariance matrix is then plotted as a variance map.

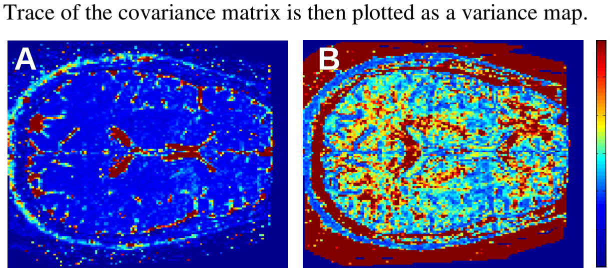

Figure 1: *Synthetic data. MC simulations with Rician noise yield the Euclidean (A) and the Log-Euclidean (B) variance maps. The Euclidean map depends on SNR and resembles an ADC map.* ***The Log-Euclidean map over-estimates the variance in anisotropic white matter.***

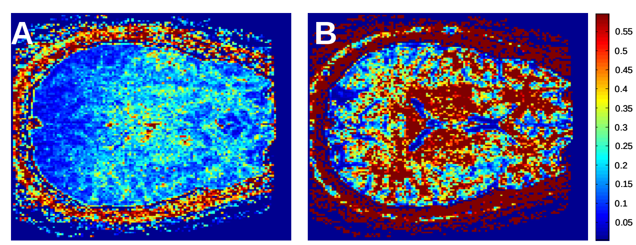

Figure 2: *MRI data. 8 repetitions of a DTI experiment yield the Euclidean (A) and the Log-Euclidean (B) variance maps. The Euclidean map affected by the multichannel reconstruction.* ***The additional noise is less dominant in the Log-Euclidean map than the over estimation of variance in anisotropic white matter.***

Results and Discussion

The synthetic data consists of Rician noise, homogenously spread over the entire image; this type of noise is expected in MR measurements due to thermal noise in the RF receiver. The Euclidean variance map (Fig. 1a) for the simulated data resembles the ADC contrast; it shows high variance in CSF areas whereas white and gray matter have the same variance, which is lower than in CSF. This is in-line with noise properties of DWIs since the DW signal in CSF is lower, hence SNR is lower and expected variability is higher. The Log-Euclidean variance map (Fig. 1b) resembles the FA; it shows high variance in white matter areas. Since it approximates affine-invariance, the Log-Euclidean metric maps tensors with extreme eigenvalue ratios to infinity. Such tensors will be a large distance from other tensors, regardless of the type of noise. Therefore the Log-Euclidean variance map is over-estimated in anisotropic white matter voxels. The 8-repetition data introduces new types of noise, such as multi-channel reconstruction, misalignment across repetitions, non-linear motion and the effect of dynamic biological processes, which should affect the variance maps. As such both variance maps present much higher variance than in the synthesized experiment. The Euclidean variance map (Fig. 2a) depends on the location within the image; this is the effect of the multichannel reconstruction, which provides higher SNR for the periphery, which is closer to the coils. This artifact dominates the Rician noise. The Log-Euclidean map is again very similar to an FA map, suggesting that additional noise and artifacts are less dominant than the mapping of extreme eigenvalued tensors to infinity.

Conclusion

The Log-Euclidean distance is over-estimated in anisotropic white matter areas. As a result the between-repetition variability for white matter areas is biased, which makes it harder to infer statistical changes. This is not the case for the Euclidean distance, which depends on the bulk diffusion. While the affine-invariant method is mathematically appealing for tensor manipulation, the findings presented here along with recent observations regarding the affine-invariant and Log-Euclidean metrics [8-9] lead us to the conclusion that the Affine-invariant and Log-Euclidean metrics are not appropriate for the analysis of the variability of diffusion tensor MRI data.

References:*[1]* Basser and Pajevic, IEEE-TMI, 22(7), 2003; *[2]* Moakher, SIAM J. Matrix Anal. Appl., 24(1), 2002; *[3]* Pennec, JMIV, 25(1), 2006; *[4]* Fletcher and Joshi, Signal Processing, 87(2), 2007; [5] Arsigny et al., MRM 56, 2006; *[6]* Commowick et al.., MICCAI 2008. *[7]* Fillard et al., IEEE-TMI, 26(11), 2007; *[8]* Pasternak et al., MMI workshop at MICCAI 2008. *[9]* Whitcher et al., MRM 57, 2007.