Iterative Soft-Thresholding Reconstruction for Time-of-Flight MR Angiography

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Introduction:

Time-of-Flight MR angiography (TOF-MRA) is a non-invasive technique that utilizes the signal enhancement due to blood inflow effects for imaging the blood vessel anatomy. Vascular images are inherently sparse and can be made even sparser in a suitable transform domain (e.g. wavelet transform), making MR angiography a natural application of Compressed Sensing (CS) [1]. There are two classes of algorithms in the current literature that are being commonly used for CS reconstruction problems: (1) greedy or matching pursuit based methods[2], (2) iterative methods based on minimizing cost functions that involve several non-linear components [3,4] (L1-norm, Total Variation, etc)and the L2-norm error. Matching pursuit based strategies are computationally expensive for clinically relevant data sets that may require a few thousand basis functions for accurate representation. On the other hand, iterative methods with non-linear cost functions may require complex derivative evaluation and are also sensitive to the choice of free parameters (or weights) associated with the non-linear terms. In this paper we present a study to develop a TOF-MRA application using a robust reconstruction method based on iterative soft thresholding with slowly decreasing thresholding parameter. This approach avoids computationally expensive cost function evaluation and does not have any free parameters. The computations in this approach involve only FFTs and wavelet transforms both of which allow fast implementation.

Methods:

Data acquired from a multi-channel MR imaging system may be represented as $y_j = \Phi_{FT}\Phi_{CSP,j}x$ where y_j is a vector of k-space measurements from the j-th coil, x is the object, Φ_{FT} the Fourier transform and $\Phi_{CSP,j}$ coil sensitivity weighting of the j-th coil. If *x* is compressible, we can introduce a basis Ψ, incoherent with Φ_{FT} and $\Phi_{CSP,j}$: $y_j = \Phi_{FT}\Phi_{CSP,j}\Psi s = \Phi_M s$. The number of measurements Y={y_1,…,y_J} may be greatly reduced from the Nyquist rate, if the representation *s* of the image in the Ψ domain is sparse. The task is to recover *s*, the coefficients of *x* projected onto the basis Ψ. We reconstruct the image by applying the following iterative loop: $s^{n+1} = T_{\lambda(n)}\{ s^n + \Phi_M^H (y - \Phi_M s^n)\}$, where Φ_M^H is the backprojection operator and the soft-thresholding operator $T_\lambda\{ s\}$ is given by: $T_\lambda\{ s\} = (|s| - \lambda)_+ \exp[i \arg(s)]$. Daubechies et al [5] have studied the iterative soft thresholding method with fixed thresholding parameter λ and have shown that the corresponding solution converges to the L1-norm solution for minimizing the cost functional: $\| y - \Phi_M s\|_2 + 2\lambda \| s\|_1$. In our adaptation we start with a relatively large initial value $\lambda(0)$ for the thresholding parameter and slowly decrease it as a function of the iteration number n, (e.g. $\lambda(n) = \lambda(0)/n$). The initial value $\lambda(0)$ is typically selected to be of the order of the highest coefficient in $\Phi_M^H y$ as computed during the first loop assuming the starting guess $s^0 = 0$. The initial choice of $\lambda(0)$ is thus data dependent. The iteration is stopped when the L2-norm error $\| y - \Phi_M s\|_2$ reduces below a pre-set threshold.

Results:

We describe two experiments with 3D TOF MR data to illustrate image reconstruction using iterative soft thresholding. All data was collected on normal volunteers who gave informed consent on a GE Signa HDx 3T scanner with 8-channel brain coil using a flow-compensated gradient-echo sequence. We acquired a fully sampled TOF data set. This data was synthetically undersampled and images were reconstructed using both soft-thresholding and conjugate-gradient based method (using wavelets and total variation priors). The reconstructions for undersampling factor of 4 with soft thresholding and conjugate gradient methods are shown in Fig. 1 (b), (c) respectively. Fig. 1 (a) shows the corresponding reconstruction with the fully sampled data set for comparison, which validates the two reconstructions in Fig. 1 (b), (c) (~ 10% L2-norm error). The k-space trajectory used for 4-fold undersampling of the data is shown in Fi.g. 1(d). The computational speed (in a MATLAB implementation) for iterative soft thresholding approach is approximately 2X faster than for the conjugate gradient approach

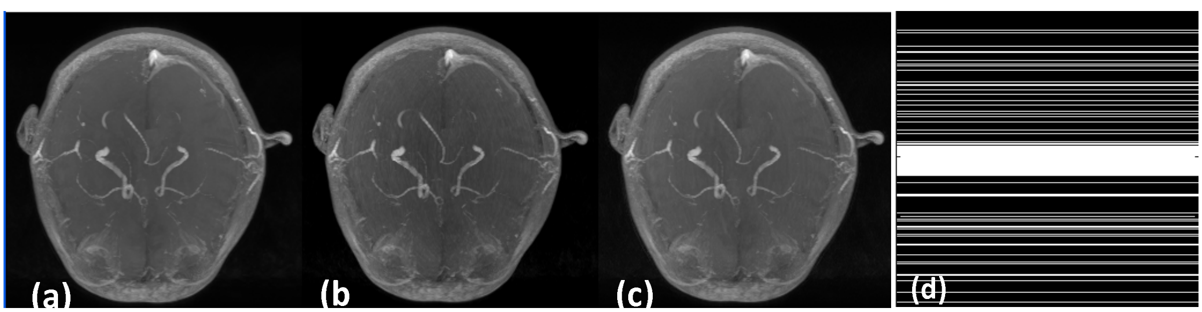

Fig.1: Maximum Intensity Projection images for 3D TOF head scan data. Reconstructions for (a) fully sampled data, (b) iterative soft thresholding method, (c) conjugate gradient method with wavelet and total variation regularization; (d) k-space trajectory used for 4-fold synthetic undersampling of the data.

Discussion:

We have presented a TOF-MRA study with CS reconstruction using an iterative soft thresholding approach. An important aspect of this method is that the computational effort essentially involves only FFT and wavelet-transform operations and no complex cost functions are involved. Further the initial choice of the thresholding parameter is data dependent making the reconstruction less sensitive to particular choice of free parameters. This simple implementation of CS using iterative soft thresholding may be used either for reducing the scan time or for improving the spatial resolution of TOF-MRA images for a fixed scan time.

References:

[1] Candes et al., IEEE Trans. Info. Theory 52, 489 (2006).[2] Tropp et al., ICASSP 2005 [3] Lustig et al. Magn.Reson. Med. 58, 1182 (2007). [4] S. Boyd, L. Vabdenberghe, *Convex Optimization* Cambridge Univ.Press(2004). [5] Daubechies et al, Comm. Pure Appl. Math 57, 1413 (2004). [6] K.King, Proc. 16th ISMRM, 1488 (2008)