First Order Catalyzing of the non-CPMG Sequence.

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Introduction

The non-CPMG sequence [1] is a spin echo sequence that permits to obtain a full magnitude signal even in the presence of initial phase variation. It employs a quadratic phase modulation of the refocusing pulses train that, seen in an appropriate frame, is equivalent to a (large) linear stationary system. It then suffices to put the spin system in an adequate state (one of the eigenstate of the enlarged system) to obtain either a constant signal or an alternating sign signal depending on the initial position of the magnetization. An efficient numerical algorithm can determine this eigenstate, but the determination of the series of pulses permitting to attain this state (catalyzing) has to rely on classical optimization procedure, which turns out to be rather imprecise. We show here that at least for nutation angle above 145°, both the eigenstate and the series of catalyzing pulses can be found analytically.

$$x_{i+1} = cZw_i x_i - jsy_i,\ y_{i+1} = -jsx_i + cZ^{-1}\overline{w}_i y_i \qquad (1).$$

$$c = \cos(\theta/2), s = \sin(\theta/2), Z = \exp(j\omega T), w_i = e^{j\delta_i}$$

$$x_{i+1} + x_{i-1} = c(Zw_i + Z^{-1}\overline{w}_{i-1})x_i,\ x_0 = \frac{1}{\sqrt{2}}, \mathrm{x}_1 = \frac{\mathrm{s}}{\sqrt{2}} \qquad (2).$$

$$\delta_i = i\Delta + \Delta/2 + \pi/2, \qquad (3).$$

$$x_{i+1} - x_{i-1} = jce^{j\Delta/2}(Ze^{j\Delta i} - Z^{-1}e^{-j\Delta i})x_i \qquad (4).$$

$$x_i = (-j)^i u_0(\omega T + i\Delta) \quad u_0(\omega T) = \sum U_k Z^k \qquad (5).$$

$$2jU_k \sin k\Delta = ce^{j\Delta/2}(U_{k+1} + U_{k-1}) \qquad (6).$$

$$u_0(\omega T) = \frac{1}{\sqrt{2}} + c(U_1^1 Z^{-1} + U_{-1}^1 Z) + c^2 \ldots \qquad (7).$$

$$U_1^1 = U_{-1}^1 = \frac{1}{\sqrt{2}} \frac{\exp(j\Delta/2)}{2j\sin\Delta} \qquad (8).$$

$$\mathrm{x}_{2\mathrm{p}} \approx \frac{1}{\sqrt{2}} + c(Z(w_{2p-1} + \ldots w_1) + Z^{-1}(\overline{w}_{2p-2} + \ldots \overline{w}_0)) \qquad (9).$$

$$\overline{w}_0 + \overline{w}_2 = w_1 + w_3 = \frac{\exp(j\Delta/2)}{2\sin\Delta} \qquad (10).$$

$$\delta_0 = -\varphi - \Delta/2, \delta_1 = -\varphi + \Delta/2, \quad \delta_2 = \varphi - \Delta/2,$$

$$\delta_3 = \varphi + \Delta/2, \quad \cos\varphi = \frac{1}{4\sin\Delta} \qquad (11).$$

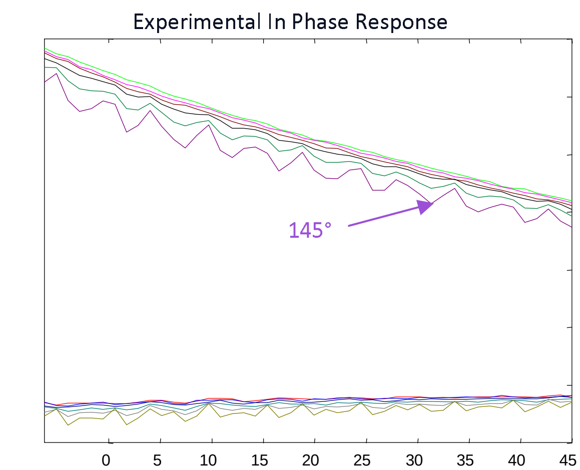
$$\delta_{0,3} = (-1.9, -0.7, 0.7, 1.9),\ \delta_{4,5,\ldots} = 2.1708, 3.3708, \ldots \qquad (12).$$

Theory

Following a procedure depicted in [1], it is always possible to write an arbitrary phase modulation in the form of a frequency sweep which, when using spinor notation, leads to the recursion (1). We will prefer to use here the second order, one component equation of movement (2), which can be obtained by combining two consecutive equations (1). (A similar equation exists for the second component *y*). In the special case of a quadratic phase modulation, the frequency sweep is linear and can be put in the form (3) leading to the equation (4). This equation has particular solutions which consist in the translation (and multiplication by a constant, here *'-j'*) of a stable function u_0 . We develop this function in form of a polynomial in the variable *Z* in (5). Considering (4), it is found that the coefficient U_k of this development must verify the equation (6). Solving (6) by brute force is not very enlightening, but decomposing it in successive power of the variable *c* (which is nominally zero), as written in (7), permits to find without too much difficulty the analytical expression up to the third order. In this abstract we stop at first order and find the equation (8) for the corresponding coefficients. It is found that this simple analytical model is quite valid down to 160°, and even acceptable at 145°. Now, considering the development, in successive power of *c,* of the equation of movement (2), one finds that the first order approximation is in the form (9) (for even echo) . One way to catalyze the response is to try to match at first order both the target distribution (the eigenstate (7)) and the current response (9). One can render the coefficients in (7) and (9) equal by using at least the four first pulses of the train, starting the linear frequency sweep (3) not before the fifth pulse. The condition is given in (10), the solution in (11). We also give in (12) the numerical values for a sweep coefficient equal to Δ=1.2 radians.

Experimental Verification

In order to assess the validity of this simplified stabilization we acquired the signal generated by a 4cm sphere (tennis table ball) filled with copper sulfate solution (500mg/l), with approximated $T_1=T_2=800$ms, using an experimental Diffusion Weighted imaging sequence [2], zeroing the selection gradient during the refocusing pulse, but keeping the crusher gradient and read gradient (the 90° and, DW preparation 180° were kept selective, with no DW gradient employed). We acquired 90 echoes, the echo spacing being 6.5ms. The TR was lengthened to 6 seconds. The Fourier transform along the read direction was performed, the signals coming from the central portion of the sphere isolated and summed. From that we found the ' In Phase component' by the sum of two successive echoes signals and the 'Out Of Phase component', obtained by the difference. The Figure 1 shows the complex In Phase Response for nutation of 175° down to 145° by step of 7.5°.



Figue 1

Discussion

The proposed catalyzing sequence cannot compete with the ones obtained by non linear programming as in [1], but its implementation simplicity may be of interest when for some reasons, it is certain that the nutation angle is above 160°. But the main interest is theoretical as it shows that the type of symmetrical spinor needed by nCPMG can be attained with good precision in a short period of time. The next step is of course to try to develop and stabilize down to the second order, in which case the solution will probably be competive with the non-linear optimization solutions.

References

[1].Le Roux,P*, J.Mag.Res*.155,278-292,2002 [2] Oner A.Y. et al. *AJNR*, 28: 575-580, 2007