Cartesian Quasi-Random Sampling for Multiple Contrasts and Dynamic Imaging

P. Ehses[1], N. Seiberlich[2], M. Blaimer[3], F. Breuer[3], W. R. Bauer[4], and P. M. Jakob[1,3]

[1]Dept. of Experimental Physics 5, Universität Würzburg, Würzburg, Germany, [2]Department of Radiology, Case Western Reserve University and University Hospitals of Cleveland, Cleveland, Ohio, United States, [3]Research Center Magnetic Resonance Bavaria (MRB), Würzburg, Germany, [4]Dept. of Internal Medicine I, Universität Würzburg, Würzburg, Germany

Introduction: Random numbers have been used in the past to shuffle the phase encoding line order in dynamic MRI [1]. The key idea is to move beyond the conventional approach that consecutive images of a time series are acquired separately and orderly. Instead the k-space data are continuously updated in a non-orderly fashion. Therefore, the number and temporal locations of time frames can be chosen retrospectively. However, pseudo-random numbers tend to "clump", meaning that even a small set of random numbers may contain elements with very similar (or identical) values, which effectively reduces the possible temporal resolution in randomly sampled MRI. Quasi-random numbers are designed to better obey the intended distribution, while still maintaining some random properties. The quasi-random shuffling of projections in dynamic radial MRI was recently shown [2]. We propose a simple method for the quasi-random shuffling of the phase encoding line order in Cartesian MRI and show some first results.

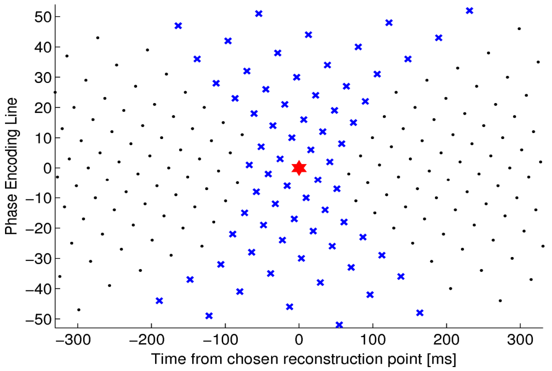

Figure 1: Gaussian weighted quasi-random sampling scheme. The lines represented by blue crosses were used for the reconstruction of the chosen time point (red hexagram).

Theory and Methods: The sequence of integers $n_k = k \cdot mod(A,B)$ (*mod* denotes the integer remaindering operation) is a permutation of the numbers k=0..B−1 if and only if A and B are relatively prime. The permutation is "most uniformly distributed" when the ratio, A/B, approximates the golden mean, in other words if A and B are consecutive Fibonacci numbers. This presents a simple and effective way to shuffle phase encoding lines in a repeated MRI experiment:

- Search for the largest Fibonacci number (f_k) that fits into the total number of TR repetitions
- Create a quasi-random shuffled series with f_k and its precessor f_{k-1} according to the above equation
- Map this shuffled series on the series of phase encoding lines (by using a distribution of choice)

It is desirable in most cases to sample the center more frequently than the outer part of k-space. To accomplish such an acquisition, we weighted the uniformly distributed quasi-random number sequence by a Gaussian distribution (illustrated in figure 1). IR-TrueFISP measurements were performed on a healthy volunteer at 1.5T with a 12-channel head-coil. A total of 2592 shuffled lines were acquired during the IR-experiment in a single-shot (TR=3.2ms, matrix=192x108, total time=8.3s). A total of 50 timeframes were reconstructed by a combination of view-sharing and parallel imaging. A variable window size, depending on the k-space-position, was chosen so that the Gaussian weighting was effectively reversed and the undersampling pattern was relatively uniform (min./max window size=42/261 TRs). To remove the remaining undersampling due to missing phase encoding lines, a GRAPPA reconstruction [3] with two different kernels (acceleration factors 2 or 3; on average 1.55) was performed. As a reference, a conventional inversion recovery TrueFISP experiment was performed with 64 time frames, acquired in 4 segments (relaxation delay after each segment: 10s, total time=1 min). T1- and T2-maps were obtained from a 3-parameter fit of the images according to [4].

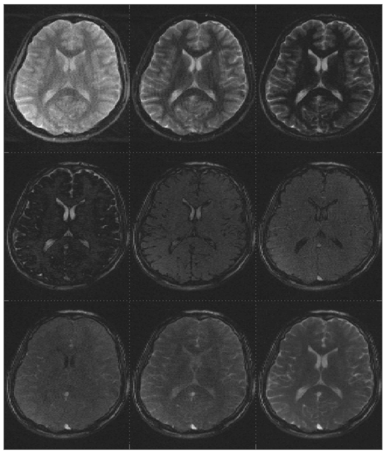

Figure 2: Representative time frames from the single-shot IR-TrueFISP experiment.

Results: A sample of reconstructed time frames of the quasi-random inversion-recovery TrueFISP experiment is shown in figure 2. Figure 3 shows the T1 and T2 maps obtained from the three parameter-fit for the quasi-random and the reference experiment.

Discussion: The key element of the proposed sampling scheme is the quasi-random shuffling of the phase encoding line order. Since it is beneficial in most cases to update the low-frequency parts of an image more frequently, it is necessary to weight the quasi-random sequence accordingly. In addition to the presented reconstruction method, the proposed acquisition scheme can be combined with other, more sophisticated reconstruction techniques. We believe that a 3D implementation of quasi-random sampling would be well-suited for compressed sensing due to the incoherency of artifacts in undersampled time-frames.

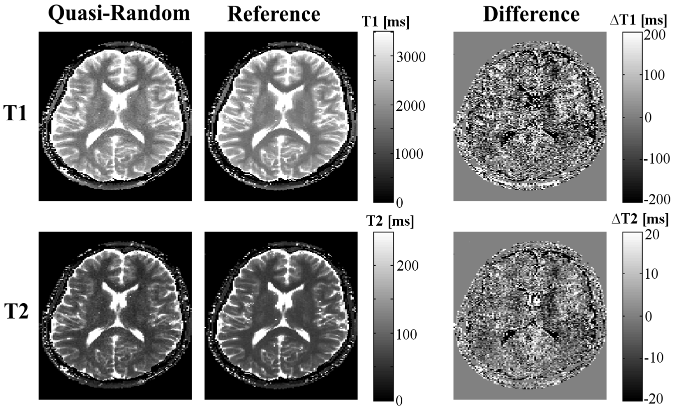

Figure 3: T1 and T2 maps obtained from the quasi-random TrueFISP measurement (left), the reference experiment (middle), and their differences (right). Note that the quasi-random measurement acquired only 1/4 of the data and took 1/8 of the scan time compared to the reference (due to the required long relaxation delay between segments).

References:

[1] Parrish T and Hu X. *Magn Reson Med.* 1995;33:326., [2] Winkelmann S, et al. *IEEE Trans Med Imaging.* 2007;26(1):68., [3] Griswold MA, et al. *Magn Reson Med* 2002;47:1202. [4] Schmitt P, et al. *Magn Reson Med.* 2004;52(3):698.