Simple harmonic oscillator based reconstruction and estimation for three-dimensional q-space MRI

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INTRODUCTION

Based on the idea that water diffuses preferably along the axonal trajectories, diffusion-weighted MRI can be used to map the neural connections between different regions of the human body. Although most methods developed to map anatomical connections employ data at low-b values, there has been recent interest to extend the acquisition scheme to larger diffusion weightings to increase the accuracy and reliability of the connectivity information [1,2]. Development of new analytical and algorithmic approaches based on single-shell high angular resolution diffusion imaging (HARDI) acquisitions has resulted in significant improvements in the quality of the reconstructed fiber orientation maps in recent years. In this study, we present a new method that maps the orientational preference of water diffusion from data acquired at several shells in q-space.

THEORY & IMPLEMENTATION

The three-dimensional q-space MR signal attenuation $E(\mathbf{q})=S(\mathbf{q})/S_0$ can be expanded in an orthogonal basis that appears in the three-dimensional quantum mechanical harmonic oscillator problem, i.e., $E(\mathbf{q}) = \sum_{N=0}^{N_{\max}} \sum_{\substack{\{l+2j=N+2\} \\ j\geq 1,\, l\geq 0}} \sum_{m=-l}^{l} A_{jlm} \Phi_{jlm}(u,\mathbf{q})$, where the basis functions are given by

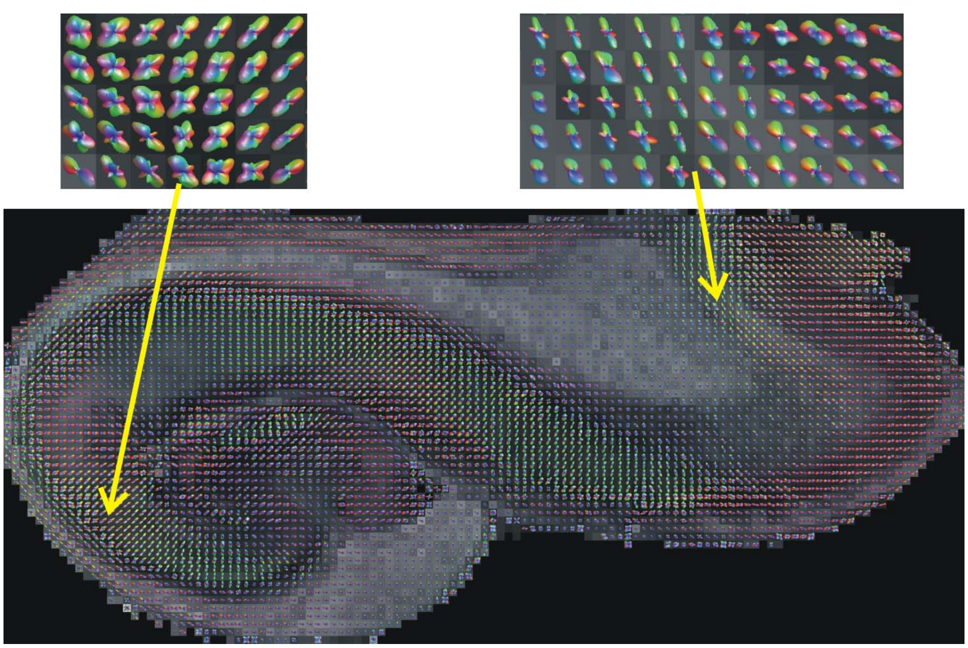
$\Phi_{jlm}(u,\mathbf{q}) = i^{-l}\sqrt{4\pi}\left(2\pi^2u^2q^2\right)^{l/2} e^{-2\pi^2u^2q^2} L_{j-1}^{l+1/2}\left(2\pi^2u^2q^2\right) Y_{lm}(\hat{\mathbf{q}})$. This is a three-dimensional extension of a scheme developed to represent one-dimensional q-space data [3]. Here $L_{j-1}^{l+1/2}(.)$ and $Y_{lm}(.)$ are the associated Laguerre polynomials and spherical harmonics, respectively, and u is a constant estimated from the data at each voxel location. The inverse Fourier transform of the $E(\mathbf{q})$ data yields the average propagator, $P(\mathbf{R})$, whose orientational dependence provides the desired connectivity information. Since the Fourier transform is linear, the propagator has an expansion with the same coefficients A_{jlm}. Moreover, the orientation-dependent radial moments defined by $\langle R^n \rangle(\hat{\mathbf{R}}) = \int_0^\infty P(R,\hat{\mathbf{R}}) R^{2+n} \mathrm{d}R$ can also be expressed with the same coefficients, where the corresponding basis functions are given by $\Psi_{jlm}(u,\mathbf{q}) = Y_{lm}(\hat{\mathbf{R}}) u^n \frac{2^{n/2}\Gamma(j+(l+n+1)/2)}{\pi (j-1)!}$.

The estimation of the A_{jlm} coefficients can be cast as a convex quadratic programming problem subject to inequality constraints. The constraints are based on the condition that the probabilities are nonnegative over a large domain in the displacement space. An additional constraint was imposed to ensure that the integral of the probability over this domain is less than unity.

RESULTS

We illustrate the performance of our approach on an excised human hippocampus data set. The data set contains a series of 330 diffusion-weighted images. The imaging was performed using a 14.1T narrow-bore spectrometer where a pulsed gradient spin echo pulse sequence was employed. The imaging parameters were: TE=29.9ms, TR=1000ms, resolution=(150x150x300)μm^3, matrix size=(134x64x18), number of repetitions=6, diffusion gradient pulse duration (δ)=2.4ms, and diffusion gradient separation (Δ)=17.8ms. The b-values were 150, 1250, 3000, 4700 and 7100 s/mm^2, where the innermost shell was sampled at 6 points and the other shells were sampled at 81 directions obtained from tessellations of an icosahedron.

The figure illustrates the map of second-order radial moments overlaid on the Fractional Anisotropy image. Despite the very high resolution of the image, crossing fiber structures were identified in several regions of the human hippocampus. Two such regions were magnified.



DISCUSSION & CONCLUSION

In this study, we demonstrated a scheme that employs arbitrary sampling of q-space possibly including data points at high b-values. In the estimation of the coefficients we exploited the positivity of the reconstructed probabilities as *a priori* information, which contributed to the robustness of the probability estimations. Note that in generating the results we have not employed any sort of smoothing other than the implicit smoothing due to the termination of the summation at order N=6. We determined this termination criterion by computing the condition number of the design matrix associated with the sampling scheme.

REFERENCES

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