

Using Spatial Priors For Better Reconstruction of Fiber Orientation Function in Diffusion MRI

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INTRODUCTION

MR Diffusion Imaging is an important noninvasive method for probing the white matter connectivity of the human brain. Current methods such as diffusion tensor imaging (DTI), high angular resolution diffusion imaging (HARDI) (1), and diffusion spectrum imaging (DSI) (2) are limited by low spatial resolution, long scan times, and low signal-to-noise ratio (SNR). These methods perform reconstruction on a voxel-by-voxel level, effectively discarding the natural coherence of the data at different points in space. We propose a Bayesian reconstruction to exploit *a priori* constraints about the smoothly varying orientation structure of white matter tracts over 3D space, and thereby improve their spatial resolution and noise tolerance.

METHODS

Let us collect the raw q-ball HARDI data from all diffusion directions in vector \mathbf{e}_i for the i th voxel. The task in q-ball reconstruction is to obtain an estimate of the orientation distribution function (ODF) \mathbf{f}_i . In (3) this was done by modeling the ODF as a sum of spherical harmonic basis functions. Let $\mathbf{Z}_Q, \mathbf{Z}_U$ be matrices of spherical harmonic functions evaluated at the measured and reconstructed directions, resp (see ref 3). Then the harmonic coefficients of the ODF are estimated by $\boldsymbol{\eta}_i = \mathbf{Z}_Q^\dagger \mathbf{e}_i$, and the ODF is then reconstructed by $\mathbf{f}_i = \mathbf{Z}_U \mathbf{P} \boldsymbol{\eta}_i$. We now propose joint estimation of HARDI ODFs simultaneously over the entire 3D space of the brain. $\mathbf{Z}_Q, \mathbf{Z}_U, \mathbf{P}$. Since the estimation is now over all the voxels enumerated as $1, \dots, p, \dots, N$, let us define the joint quantities

$$\boldsymbol{\eta} = [\boldsymbol{\eta}_1^T, \dots, \boldsymbol{\eta}_p^T, \dots, \boldsymbol{\eta}_N^T]^T, \mathbf{f} = [\mathbf{f}_1^T, \dots, \mathbf{f}_p^T, \dots, \mathbf{f}_N^T]^T$$

$$\overline{\mathbf{Z}}_Q = \begin{bmatrix} \mathbf{Z}_Q & & \\ & \ddots & \\ & & \mathbf{Z}_Q \end{bmatrix}, \overline{\mathbf{Z}}_U = \begin{bmatrix} \mathbf{Z}_U & & \\ & \ddots & \\ & & \mathbf{Z}_U \end{bmatrix}, \overline{\mathbf{P}} = \begin{bmatrix} \mathbf{P} & & \\ & \ddots & \\ & & \mathbf{P} \end{bmatrix}$$

So that $\hat{\mathbf{f}} = \overline{\mathbf{Z}}_U \hat{\boldsymbol{\eta}}$, and the task is to estimate the $\hat{\boldsymbol{\eta}}$ that simultaneously fits both the diffusion measurements within each voxel (data consistency) and gives low value for a smoothness cost function that encodes prior spatial constraints. We propose the following algorithm:

Begin with $\boldsymbol{\eta} = \boldsymbol{\eta}_0$. Then for $k = 1$ to K , repeat :

$$\hat{\boldsymbol{\eta}}^k = \arg \min_{\boldsymbol{\eta}} \left\{ \|\mathbf{e} - \overline{\mathbf{P}} \overline{\mathbf{Z}}_Q \boldsymbol{\eta}\|^2 + \lambda^2 \|\boldsymbol{\eta}\|^2 + \mu^2 \|\mathbf{D} \mathbf{W}(\boldsymbol{\eta}^{k-1}) \boldsymbol{\eta}\|^2 \right\}$$

$$\text{where } \mathbf{W}(\boldsymbol{\eta}) = [w_{p,q}], w_{p,q} = 1 - \frac{\|\boldsymbol{\eta}_p - \boldsymbol{\eta}_q\|}{\|\boldsymbol{\eta}_p\| + \|\boldsymbol{\eta}_q\|}$$

where the first term in the cost function enforces fidelity to observed data \mathbf{e} , and the second term enforces Tikhonov regularization. The third term is new, and introduces a spatial smoothness cost. Matrix \mathbf{D} is a first difference operator, and $\mathbf{W}(\boldsymbol{\eta})$ is a matrix that represents a neighborhood weighting function. If $\mathbf{W}(\boldsymbol{\eta}) = \mathbf{I}$, the globally smooth solution results. When it is updated using the current estimate of the spherical harmonics as in above, it weighs neighboring voxels in inverse proportion to the similarity of their coefficients and hence prevents indiscriminate smoothing of ODFs across possibly discontinuous fiber boundaries. We have proposed a simple iterative scheme whereby both $\mathbf{W}(\boldsymbol{\eta})$ and $\boldsymbol{\eta}$ are updated alternately.

PRELIMINARY STUDIES

Noisy HARDI data was simulated from fiber tract configurations arranged in 3D. Three performance criteria were evaluated: root mean square error, error of generalized fractional anisotropy, and fiber orientation accuracy, and shown in fig 1. Neither the original spherical harmonic q-ball method (shQBI, 3) nor post-processing by applying a 3x3x3 box smoothing filter is able to achieve the level of performance of the proposed spatially-constrained technique. In vivo reconstructed ODFs are in fig 2. The difference is especially visible at voxels with crossing fibers, as pointed out by arrows.

REFERENCES

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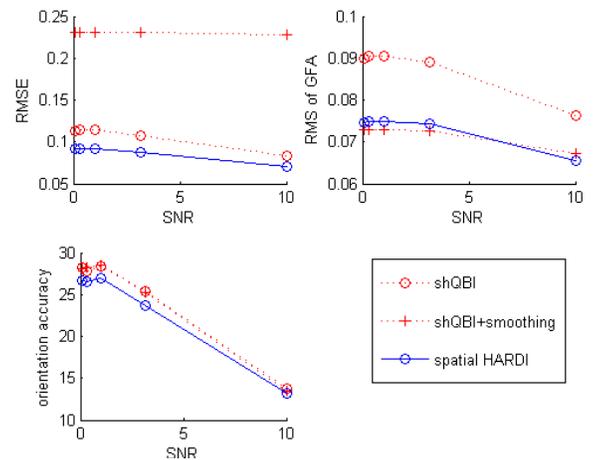


Fig 1: Performance versus SNR, order 4, $\lambda = 0.2$. $\mu =$

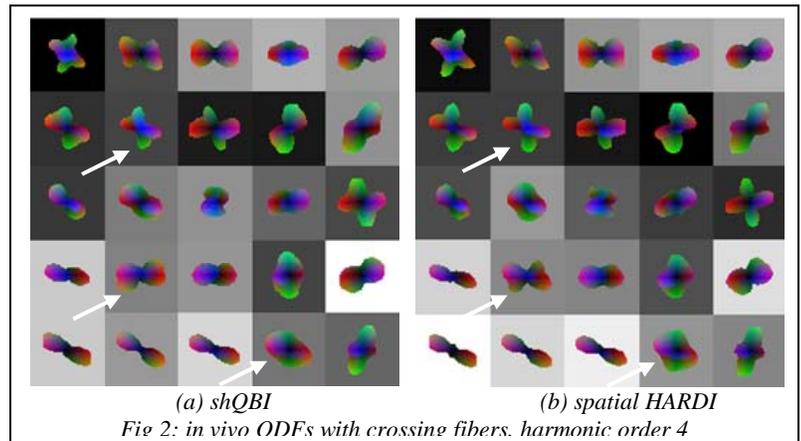


Fig 2: in vivo ODFs with crossing fibers. harmonic order 4