Efficient and Effective Anisotropic Smoothing of Diffusion Tensor Images in Log-Euclidean Framework

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Introduction

Magnetic resonance diffusion tensor imaging provides a 3×3 2nd order tensor of water diffusion based on acquired diffusion weighted images (DWI). Eigenvectors of the diffusion tensor can be exploited to characterize tissue micro-structure and architecture. However, diffusion tensor images (DTI) usually have poor signal-to-noise ratio (SNR) because they use single-shot echo-planar imaging sequences. High image noise causes problems such as erroneous calculation of the principal diffusion direction (PDD) (2). To reduce the noise, we have developed a non-iterative anisotropic filtering algorithm for smoothing DTIs. The performance of this method has been evaluated on human 3D data acquired at 3T.

Method

1. Log-Euclidean framework for tensor computing

Diffusion tensors (symmetric positive definite matrices) constitute a manifold instead of a vector space, so tensor computing in a Euclidean framework may cause some tensors to go out of the manifold. Thanks to a recently proposed Log-Euclidean framework (3), tensor processing can be converted into simple and efficient Euclidean computing for vectors by transforming the tensor manifold to tensor logarithm space. A diffusion tensor **D** is mapped to **L** in tensor logarithm space by **L** = log (**D**) (Eq. 1), while **L** can be inversely transformed back to the tensor manifold by **D** = exp (**L**) (Eq. 2) after being processed.

$$L = log(D) = V\begin{bmatrix} log(d_1),0,0 \\ 0,log(d_2),0 \\ 0,0,log(d_3) \end{bmatrix}V^T \text{ where } D = V\begin{bmatrix} d_1,0,0 \\ 0,d_2,0 \\ 0,0,d_3 \end{bmatrix}V^T \quad \textbf{(1)} \quad D = exp(L) = U\begin{bmatrix} exp(l_1),0,0 \\ 0,exp(l_2),0 \\ 0,0,exp(l_3) \end{bmatrix}U^T \text{ where } L = U\begin{bmatrix} l_1,0,0 \\ 0,l_2,0 \\ 0,0,l_3 \end{bmatrix}U^T \quad \textbf{(2)}$$

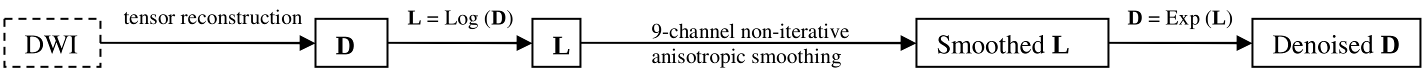
$$\frac{\partial I_m}{\partial t} = div(T\nabla I_m) \quad \textbf{(3)} \qquad G = K_\rho * \sum_m (\nabla I_m \otimes \nabla I_m) \quad \textbf{(4)}$$

2. Non-iterative anisotropic filtering for multi-channel images

The anisotropic diffusion smoothing we propose is governed by Eq.3 above, where I_m is the image intensity in channel m and ***T*** is a *structure tensor* that provides the directionality of smoothing. ***T*** is properly constructed from a common *gradient tensor* **G** (Eq. 4) so that smoothing along the boundaries is encouraged and smoothing across them is discouraged. Eqn. 3 is solved with an unconditionally stable and second order accurate semi-implicit scheme, which allows us to choose a very large step size. An optimal effect can be achieved by only one iteration, which actually turns the iterative diffusion filtering to a non-iterative one.

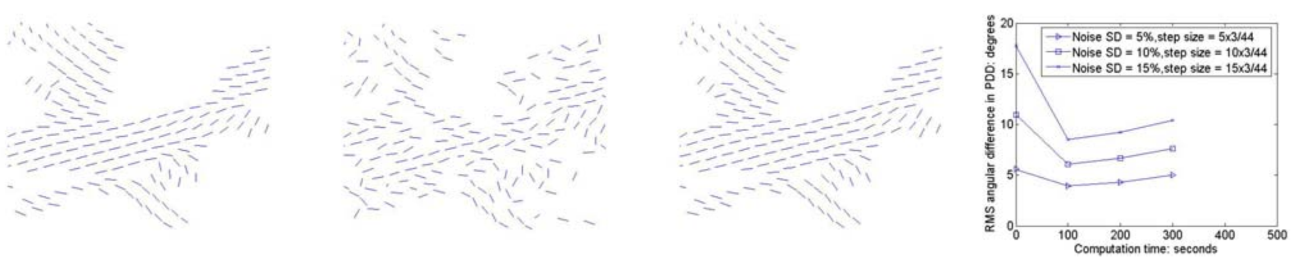
3. Outline of DTI anisotropic smoothing in the Log-Euclidean framework

To summarize, the anisotropic DTI denoising procedure can be described by the following diagram:



Experiments and Results

DTIs were acquired *in vivo* with a 3T Philips Achieva MR scanner with 32 non-collinear weighting directions (b = 1000 s/mm^2), which generated a volume of 256×256×120 mm^3 at an isotropic resolution of 2×2×2 mm^3. Ten repeated scans were co-registered and averaged to yield a "clean" volume with an SNR of ~75. A block of seven slices in this clean dataset was corrupted with zero mean Gaussian noise at standard deviation (SD) = 5%, 10% and 15% times the DWI intensity for smoothing tests. The middle slice of the test block was chosen for quantitative evaluation. The *effectiveness* of anisotropic smoothing was assessed by the root mean square (RMS) angular difference in PDD with respect to clean data, and time *efficiency* was evaluated on the basis of computation time on a COMPAQ laptop (Mobile AMD Sempron 2800). Fig. 1 (a-c) displays that disarranged PDDs have been greatly restored by one iteration of anisotropic smoothing with structure boundaries well preserved. Fig. 1 (d) shows that it takes only one iteration (~100 seconds) to gain the optimal PDD improvement of 29%, 44% and 52% for noise at SD = 5%, 10% and 15% respectively with our anisotropic filter.



(a) (b) (c) (d)

Fig. 1 (a) An enlarged view of PDDs in a region of interest (ROI) within the clean data. The line segment represents the PDD orientation. (b) PDDs in the ROI corrupted with zero mean, SD = 15% Gaussian noise. (c) PDDs in the ROI after one iteration of anisotropic smoothing. (d) The variations with computation time of RMS angular difference in PDD over the whole middle slice.

Conclusion and Discussion

We have developed a technique to denoise DTIs by performing non-iterative anisotropic smoothing in a Log-Euclidean framework. The presented method achieves great computational efficiency mainly due to the non-iterative multi-channel anisotropic filtering process. In addition, directly smoothing DTIs is more efficient than smoothing DWIs in that only 6 components need to be processed for each voxel in DTI while DWIs usually contain more than 7 channel images. However, our method is specific to a Gaussian diffusion model (i.e. tensor model) and needs to be extended to apply to non-Gaussian diffusion models.

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Reference

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