

# MR Elastography Inversions Without Phase Unwrapping

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**Introduction:** Dynamic MR elastography (MRE, [1]) is a phase-contrast MRI technique used to measure the mechanical properties of tissue in vivo. Periodic stresses are applied to the tissue, and the resulting displacement of the tissue is encoded into the phase of the MR transverse magnetization and imaged. These displacement images are then processed to provide estimates of tissue mechanical properties, such as the shear modulus. Since MRE is a phase-based technique, the displacement data typically must be unwrapped first before subsequent processing is performed [2]. For some data, this can be a slow and error-prone process. If unwrapping is not performed correctly, the displacement data are not valid, and this can cause errors in the estimates of material properties. The purpose of this work is to demonstrate an algorithm for processing MRE phase data that includes the removal of longitudinal wave and background phase effects without requiring phase unwrapping.

**Theory:** Let  $\mathbf{u}(\mathbf{r},t)$  and  $\mathbf{U}(\mathbf{r},f)$  be the time- and frequency-domain displacements that result inside an object due to a periodic stress, and assume that the material can be modeled as an infinite, homogeneous, linearly viscoelastic, and isotropic medium. The displacement field satisfies Eq. (1), where  $\rho$  is the density,  $\omega=2\pi f$ , and  $A(f)$  and  $G(f)$  are the two complex-valued, frequency-dependent Lamé constants that describe the mechanical properties of the material [3]. While the shear modulus  $G$  and shear stiffness  $\mu$  have been shown to be useful quantities for assessing diseases such as hepatic fibrosis [4-6], the constant  $A$  (related to longitudinal wave propagation) varies little between different tissues [7]. However, since it is several orders of magnitude larger than  $G$ , its presence in the equations of motion can cause difficulties in inverting them to find  $G$ . A typical approach to removing the longitudinal wave information is to take the vector curl of Eq. (1), yielding Eqs. (2) and (3), where  $\mathbf{C}$  is the curl of the original displacement field. The MRE motion-encoding process [8] results in a series of images of the phase difference of the MR transverse magnetization,  $\phi(\mathbf{r},t)$ , which are predominantly the sum of a static background term  $\beta(\mathbf{r})$  that typically includes off-resonance, gradient-imbalance, and concomitant-field phase information, and a time-dependent term  $\theta(\mathbf{r},t)$  whose frequency spectrum is proportional to that of the true motion due to the motion-encoding gradients (MEG) (i.e., Eq. (4), where  $\mathbf{m}(t)$  and  $\mathbf{M}(f)$  are the time- and frequency-domain descriptions of the MEG, and  $\leftrightarrow$  indicates a temporal Fourier transform) [8]. The Jacobian (or gradient tensor) of  $\phi(\mathbf{r},t) = \theta(\mathbf{r},t) + \beta(\mathbf{r})$  is thus  $\nabla\phi(\mathbf{r},t) = \mathbf{m}(t) \otimes \nabla\mathbf{u}(\mathbf{r},t) + \nabla\beta(\mathbf{r})$ , and the temporal Fourier transform of the Jacobian is given by Eq. (5), where  $\delta(f)$  is the delta function and a comma in a subscript indicates differentiation in that direction. The curl of the frequency-domain phase data can then be written as Eq. (6), where  $\epsilon_{hjk}$  is the Levi-Civita symbol. MRE analysis of the phase data to determine mechanical properties is performed at one or several nonzero frequencies. Therefore, the static term  $\beta_{j,k}(\mathbf{r})\delta(f)$  can be neglected. Also, if the same MEG is applied when measuring each component of motion, as is usually the case, then  $M_j(f) = M(f)$  for all  $j$ . This means that the curl of the phase data can be written as Eq. (7), which means the curl of the original phase data, even with static phase artifacts, is just proportional to the curl of the displacement data and can be readily substituted into the Helmholtz wave equation (Eq. (3)) to solve for  $G$ . The main practical challenge to implementing this algorithm is that the original phase data  $\phi(\mathbf{r},t)$  are wrapped, so derivatives and Fourier transforms of the phase data cannot be calculated unless the data are unwrapped first. However, derivatives of the phase can still be calculated even without unwrapping the phase by using the chain rule (as in [9]). By creating the unit-magnitude complex image  $p_j(\mathbf{r},t) = \exp(i\phi_j(\mathbf{r},t))$ , the derivatives of the phase data can be calculated using Eq. (8), where  $\Re$  and  $*$  are the real part and the complex conjugate of a complex quantity, respectively. The derivatives of  $p$  only involve derivatives of the real and imaginary parts of this complex quantity, and thus they do not have the same discontinuity limitations that calculating the derivative of phase data directly has. Therefore, the curl of the phase or displacement data can be calculated without having to unwrap the phase data. The curl data can then either be used by itself, or with other techniques like directional filtering [10], to invert the Helmholtz equation for the shear modulus, and the shear modulus can be converted to the shear stiffness  $\mu$  using Eq. (9), where  $|\cdot|$  is the magnitude of a complex quantity [11].

$$-\rho\omega^2\mathbf{U}(\mathbf{r},f) = (A(f) + G(f))\nabla(\nabla\cdot\mathbf{U}(\mathbf{r},f)) + G(f)\nabla^2\mathbf{U}(\mathbf{r},f) \quad (1)$$

$$-\rho\omega^2\nabla\times\mathbf{U}(\mathbf{r},f) = G(f)\nabla^2(\nabla\times\mathbf{U}(\mathbf{r},f)) \quad (2)$$

$$-\rho\omega^2\mathbf{C}(\mathbf{r},f) = G(f)\nabla^2\mathbf{C}(\mathbf{r},f) \quad (3)$$

$$\theta(\mathbf{r},t) = \mathbf{m}(t) \otimes \mathbf{u}(\mathbf{r},t) \leftrightarrow \Theta(\mathbf{r},f) = \mathbf{M}(f)\mathbf{U}(\mathbf{r},f) \quad (4)$$

$$\nabla\phi(\mathbf{r},t) \leftrightarrow \nabla\Phi(\mathbf{r},f) = \Phi_{j,k}(\mathbf{r},f) = \mathbf{M}(f)\nabla\mathbf{U}(\mathbf{r},f) + \nabla\beta(\mathbf{r})\delta(f) \quad (5)$$

$$\nabla\times\Phi(\mathbf{r},f) = \epsilon_{hjk}\Phi_{j,k}(\mathbf{r},f) = \epsilon_{hjk}(M_j(f)U_{j,k}(\mathbf{r},f) + \beta_{j,k}(\mathbf{r})\delta(f)) \quad (6)$$

$$\nabla\times\Phi(\mathbf{r},f) = M(f)\epsilon_{hjk}U_{j,k}(\mathbf{r},f) = M(f)\nabla\times\mathbf{U}(\mathbf{r},f) = M(f)\mathbf{C}(\mathbf{r},f) \quad (7)$$

$$\phi_{j,k}(\mathbf{r},t) = \Re\left\{-ip_j^*(\mathbf{r},t)p_{j,k}(\mathbf{r},t)\right\} \quad (8)$$

$$\mu(f) = \rho c_s^2(f) = 2|G(f)|^2 / \left[ \Re(G(f)) + |G(f)| \right] \quad (9)$$

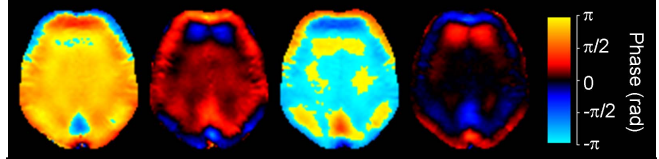


Figure 1: 4 offsets of the horizontal component of motion for 1 slice.

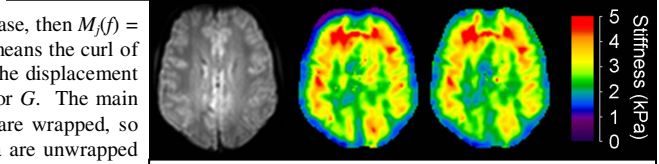


Figure 2: (a) Magnitude image and elastograms (b) with and (c) without phase unwrapping.

**Methods:** In vivo human brain MRE data were used to validate that this new processing strategy can yield elastograms equivalent to those which require phase unwrapping. Data were collected in a healthy volunteer after obtaining informed consent and in accordance with the Mayo Clinic IRB. The acquisition was performed using a 3T whole-body scanner (SIGNA HDx, GE, Milwaukee, WI) using a single-channel T/R head coil and a multislice, flow-compensated, GRE MRE pulse sequence with the following parameters: axial plane, FOV=24 cm, 80x80 acquisition matrix, TR/TE=1065/25.9 ms, 2-mm slice thickness with 1-mm gap, 45° flip angle, ±16 kHz bandwidth, 32 slices, 60-Hz motion, tetrahedral motion encoding with 1 16.7-ms flow-compensated MEG on each axis [12] with an amplitude of 1.6 G/cm. The phase data were processed with and without phase unwrapping to obtain the curl, which was then input into a direct inversion algorithm using 20 3D directional filters (4<sup>th</sup>-order Butterworth radial bandpass filters with cut-off frequencies of 0.001 and 40 cycles/FOV) to estimate the shear stiffness of the tissue.

**Results:** Figure 1 shows the 4 phase images for the center slice and the horizontal motion-encoding direction. The presence of large shifts in the phase data in offsets 1 and 3 resulted in the phase data being centered near  $\pi$  and  $-\pi$ . While in this case a standard 2D algorithm can unwrap each offset by itself, making sure that there are not factors of  $2\pi$  that differ between the offsets and between the other slices can be challenging and time consuming absent a true 3D or 4D unwrapping algorithm. Figure 2 shows the magnitude image for the center slice and the elastograms obtained with and without phase unwrapping. The two results agree well with each other.

**Discussion and Conclusions:** These results demonstrate that MRE stiffness inversions without phase unwrapping are feasible and that these results are comparable to those that would be obtained normally. Since phase unwrapping can be time-consuming and error-prone, avoiding the process altogether can be quite beneficial. This technique offers promise in applications where the wave data are too complicated and/or the wave amplitude is too high to be unwrapped correctly (e.g., in the brain), background phase terms are large enough to cause unwrapping problems (e.g., oblique tetrahedral acquisitions in muscle), and SNR problems cause regions with low signal to affect the accuracy of phase unwrapping elsewhere in the images (e.g., abdominal MRE in the vicinity of the stomach and bowels).

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