A Kalman filtering framework for prospective motion correction

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Introduction: Prospective motion correction is becoming a feasible means of overcoming the problem of patient motion in brain imaging. Navigator data from a tracking system is used to update the scanner gradients, and therefore the position of the imaging volume, before every spin excitation. Regardless of the tracking system used, accuracy is a critical factor as any position noise results in image artefacts [1]. Thus, position errors must be minimised. Due to latency in the system, one should also predict the position of the object at a certain time, Δt, in advance. Finally, an estimate of residual position errors would be useful for post-processing correction. The goal of this work was to develop a pose prediction method based on Kalman filtering that reduces tracking noise and enables retrospective estimation of residual errors. The method was validated using simulated and experimental pose data from an optical tracking system (similar to that described in [2]).

Method: The tracking system provides position and orientation information in six degrees of freedom (6DOF); the Kalman filter, predictor and smoother (Fig. 1) are applied to each DOF separately.

The filtering problem involves finding an estimate, $\hat{x}_t$, of the true signal state, x_t, given noise-corrupted measurement data, $z_t = x_t + \text{noise}$. The Kalman filter [3] is one such approach and is optimal in a mean squared error sense. Here, we use a position-velocity (PV) model, meaning that for each DOF the state contains position and velocity information (Fig. 2a). Following a standard Kalman filtering approach [4], the initial estimate of the next state is given by $\hat{x}_t^- = A\hat{x}_{t-1}$, where A is the transition matrix. Given the latest measurement, z_t, the updated estimate of the state is then given by $\hat{x}_t = \hat{x}_t^- + K_t(z_t - H\hat{x}_t^-)$, where K is the Kalman gain (Fig. 2d) and H is the measurement matrix (Fig. 2b), indicating that position is the measured parameter, not velocity.

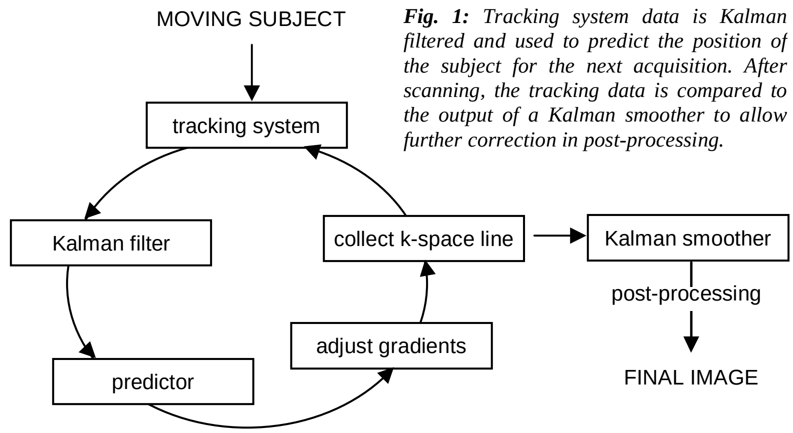

Fig. 1: Tracking system data is Kalman filtered and used to predict the position of the subject for the next acquisition. After scanning, the tracking data is compared to the output of a Kalman smoother to allow further correction in post-processing.

Given the estimate $\hat{x}_t$ at time t, the state value at time point $t + \Delta t$ is predicted. The current best estimate of velocity and position is used to compute $\hat{x}_{t+\Delta t}$ (Fig. 2b) for the latency time, $\Delta t = 33\,\text{ms}$, which is approximately equal to the time taken to acquire two camera frames. Finally, a forward-backward Kalman smoother [5] was implemented to obtain the best possible estimate of subject motion after the scan, to enable correction of residual errors in post-processing. The post-processing stage is currently only implemented for in-plane translational motion and functions by correcting phase information in k-space.

When applied to real data, no 'ground truth' is available with which to compare the results. Therefore we have performed simulations with known values of x_k, but with randomly generated measurement noise.

(a) $x_t = \begin{bmatrix} p \\ v \end{bmatrix}$ (c) $\hat{x}_{t+\Delta t} = \begin{bmatrix} 1 & t_L \\ 0 & 1 \end{bmatrix} \hat{x}_t$

(b) $H = \begin{bmatrix} 1 & 0 \end{bmatrix}$ (d) $K_t = P_t^- H^T (H P_t^- H^T + R)^{-1}$

where $P_t^- = A P_{t-1} A^T + Q$

Fig. 2: (a) state representation, (b) measurement matrix, (c) predicted state, and (d) Kalman gain. P is the error covariance estimate and Q is the process variance matrix.

Results and Discussion: Fig. 3 presents results of the post-processing stage. Fig. 4a shows simulated filtering results. True position data (green) are plotted against measured data (red) and the output of the Kalman smoother (black). The mean absolute error (MAE) between the measured values and the true signal is 41 µm (a realistic value for our tracking system); this is reduced to 13 µm between the Kalman smoother output and the true signal. Thus, the smoothed output accurately estimates the true signal and is therefore suitable for using to predict residual errors.

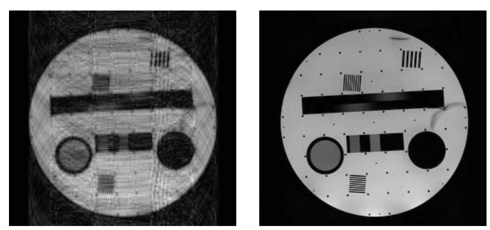

Fig. 3: Data acquired using a gradient echo sequence with erroneous position information. Left: uncorrected. Right: corrected using post-processing given knowledge of the position errors.

Fig. 4b shows real data (red) obtained from a human subject at a sample rate of 60 Hz. Values are predicted 33 ms in advance using the filter described here (blue). Kalman smoother results are shown for comparison (black). The MAE between the smoothed estimates and the predicted estimates is 25 µm. Subtracting the smoothed results from the predicted results gives an estimate of errors made during prospective correction. These data can then be used for post-processing.

Although simulations indicate that the framework presented here is effective, a rigorous quantification of its performance using a tracking target with known motion is needed. This will also allow for improved optimisation of the filter parameters. Another limitation of this work is the lack of constraints concerning the possible values for head position, velocity and acceleration. This information should be incorporated. Due to the nature of the Kalman filter, this approach could also be used to optimally combine tracking information from different tracking systems, measuring different motion parameters.

Conclusion: We have developed a pose prediction system based on Kalman filtering that reduces tracking noise and enables retrospective estimation of residual errors. This reduces the accuracy requirements of the tracking system itself.

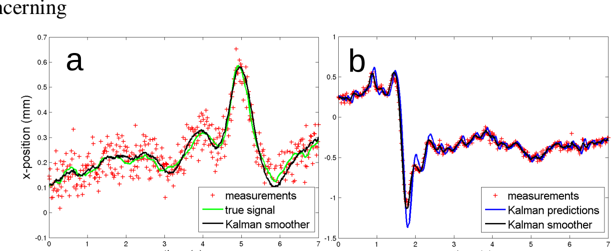

Fig. 4: (a) Kalman smoother applied to simulated data; (b) Kalman predictor and smoother applied to real data with significant motion. Note the overshoot in predicted values caused by a velocity change.

References: [1] Zaitsev et al., Proc. ISMRM, p. 3114 (2008). [2] Zaitsev et al., Neuroimage 2006;31(3):1038-50. [3] Kalman, Journal of Basic Engineering 1960;82(1):35–45. [4] Welch & Bishop, SIGGRAPH, 2001. [5] Gelb et al., Applied Optimal Estimation, 1974.

Acknowledgements: This work is a part of the INUMAC project supported by the German Federal Ministry of Education and Research, grant #01EQ0605. The authors thank Bruce Griffin and Stathis Hadjidemetriou for their helpful advice concerning Kalman filtering.