Self-adjusted regularization ratio for Robust Compressed Sensing

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Introduction Minimizing the total Variation (TV) norm term in conjunction with a fidelity term has been effectively applied to Magnetic Resonance (MR) image reconstruction with partially acquired data [1, 3]. However, an inappropriate ratio, determined by the regularization parameter, between these two terms may result in either residual artifact or reduced spatial resolution. The optimization of the predefined regularization parameter is not trivial. This work aims to make this framework more robust to the choice of the regularization parameter. A self-adjustment technique is proposed in this work to automatically optimize the ratio between these two terms. Using compressed sensing (CS) [1] as an example, experiments with both phantom and in vivo data sets demonstrated that the proposed method made the regularized reconstruction framework less sensitive to the choice of regularization parameter. This work dramatically reduces the difficulty of parameter decision and increases the practicability of regularized reconstruction techniques.

Theory Let X be the partially acquired MR data; I be the reconstructed image; Ω be the image domain; ∇ be the gradient operator; MR(·) be MR encoding operator. CS-MRI minimizes the energy functional defined by Eq. 1 for reconstruction, where λ is the regularization parameter balancing the data fidelity and regularization. In this work, it is assumed that the difference X – MR(I) is Gaussian distributed with zero mean and variance σ^2, which is to be optimized. To maximize the likelihood of this assumption, Eq. 1 is modified to be Eq. 2 through minimizing the negative log-likelihood. This model allows a to-be-optimized variance between the reconstructed data and acquired data. This coupled optimization of *I* and σ^2 makes the ratio between the regularization term and fidelity term be updated during the iteration. The ratio $2\lambda\sigma^2$ is updated in the way that when the reconstructed data gradually close to the acquired data, the weight on the regularization term automatically reduced. Using energy decent method it can be deduced that σ^2 is the standard deviation of the difference X – MR(I). To avoid singularity, a lower bound needs be defined for σ.

$$E[I \mid X] = \|X - MR(I)\|_2^2 + \lambda \int_\Omega |\nabla I| \, dx \qquad (1)$$

$$E[I, \sigma \mid X] = \frac{1}{2\sigma^2}\|X - MR(I)\|_2^2 + \lambda \int_\Omega |\nabla I| \, dx + \frac{|\Omega|}{2}\log 2\pi\sigma^2 \qquad (2)$$

Methods Two sets of experiments were used to demonstrate the performance of the proposed model (Eq. 2). The first set compared the reconstructions using Model 1 (Eq. 1) and Model 2 (Eq. 2) with regularization parameter λ from 0.005 to 0.5. The second experiment used the same regularization parameter 0.01 for 3 different applications. The implementation of CS was based on Ref [1]. Except for the GE phantom, all data sets were acquired on a 3T system (Philips, Best, Netherlands). Fully acquired k-space data sets were artificial undersampled to simulate the partial acquisition. Acceleration factor 2 was used for all experiments. Vary Density (VD) acquisition scheme was used to design the probability density function (PDF) for controlled random acquisition. CS was applied for reconstruction and no parallel imaging techniques [2] were used. The image reconstructed with full k-space data was used as reference for the calculation of root mean square error (RMSE).

Results Fig. 1 shows the results of the first experiment set. The upper row shows the results with the fixed regularization ratio (Model 1). The lower row shows the results with the self-adjusted regularization ratio (Model 2). Regularization parameter λ = 0.005, 0.05, and 0.5 was used for reconstruction of the three columns respectively. It can be seen that the proposed method generated consistent results (Figs. 1d~1f) with different λ, while the conventional method with fixed regularization ratio was sensitive to λ (Figs. 1a~1c). Quantitatively, the RMSEs of Figs. 1a~ 1c are 31.0%, 12.3%, and 24.9% respectively. On the contrary, more consistent RMSEs 13.9% (Fig. 1d), 9.0% (Fig. 1e), and 13.7% (Fig. 1f) were achieved with the proposed model. This demonstrates that Model 2 is less sensitive to the choice of regularization parameter than Model 1. The lower row of Fig. 2 shows the results of Model 2 with the fixed regularization parameter λ = 0.01 for different applications. For comparison, the upper row shows the results of direct Fourier transform with the partial k-space data. In all results, artifact reduction and resolution preservation are well balanced. This demonstrated that proposed model increase the applicability of CS-MRI.

Discussion In all experiments, the proposed model makes CS-MRI less sensitive to the choice of the regularization parameter. Moreover, 1) the reconstruction time is almost identical to that for CS-MRI with the fixed regularization ratio; and 2) this model can automatically adjust the ratio when the regularization parameter is either larger or smaller than the optimized value. Both properties are not true for Bregman iteration based method [3]. Bregman iteration takes longer reconstruction time and works only when λ is larger than the optimized value. In conclusion, a self-adjustment technique has been presented to increase the applicability of regularized reconstruction by reducing the sensitivity to the regularization parameter.

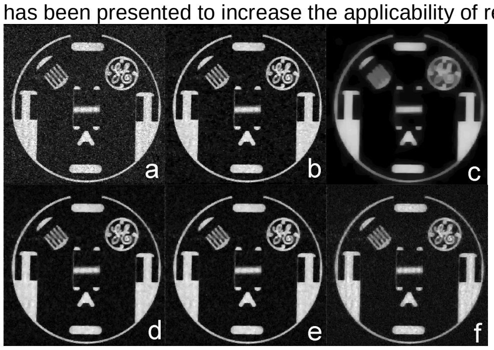

Fig. 1 Comparison of CS with fixed (a~c) and self-adjusted (d~f) regularization ratio. Three columns are results with λ = 0.005, 0.05, and 0.5 respectively.

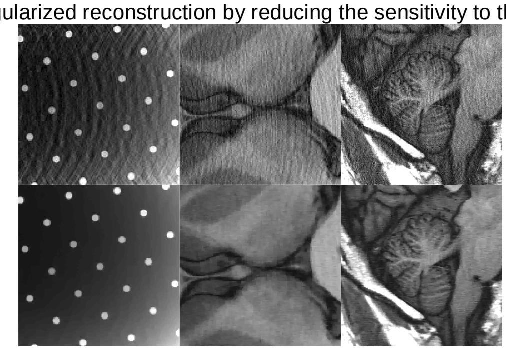

Fig. 2 Results of self-adjusted regularization ratio with fixed regularization parameter λ = 0.01. Acceleration factor was 2 for all 3 applications.

References:

[1] Lustig M et al., MRM 2007; 58:1182-1195. [2] Pruessmann K. P. et al. Magn Reson Med 1999; 42:952-962. [3] Liu B, et al., ISMRM 2008; p 13