Sparse Image Reconstruction using the Generalized Sampling Theorem for MR Angiography

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Introduction: The reconstruction of images from highly undersampled data, especially for the reconstruction of high frame rate MR Angiography images, has recently become a topic of great interest. Many methods which exploit both the sparse nature of angiographic images as well as the relatively mild undersampling artifacts of the radial trajectory have been introduced, including compressed sensing [1] and the HYPR family (for instance, [2-4]). This abstract proposes a new reconstruction method for sparse data using the Generalized Sampling Theorem (GST) [5] of Papoulis, called GST-MRA. GST implies that as long as the number of non-zero pixels in an image is less than the number of acquired k-space points, the image can be fully reconstructed. However, because the image matrix is much larger than the amount of data acquired for a given time frame, a constraint must be identified to reduce the total pixels in the final image. The composite image, made up of data from all time frames, is ideal for this purpose; by defining a threshold, a mask which limits the total number of image pixels can be created. This mask can be further sparsified by multiplying it by coil sensitivity maps. Given a very small system, the image can be solved for directly by defining and inverting the encoding function. However, in practice, this encoding matrix is large and ill-conditioned, making inversion both time-consuming and unstable. In addition, only a hard (i.e. logical) mask can be used in the direct method. For these reasons, GST-MRA has been implemented using the conjugate gradient algorithm (as in CG-SENSE [6]) in conjunction with a soft mask made up of the thresholded composite image and the coil sensitivity maps. GST-MRA is demonstrated in numerical simulations as well as in an AVM patient. Methods: The GST-MRA algorithm is shown in Figure 1. The first step is to generate a composite image using the data

from all time frames. A soft mask is created by thresholding the composite image; this mask is then multiplied by coil sensitivity maps to further sparsify it for the subsequent reconstruction. A starting image for the CG iteration is created by gridding the data for a given time frame using NUFFT [7] and multiplying the resulting image by the mask. A slightly modified CG method is then employed, in that the intermediate images are multiplied by the inverse of the mask before resampling along the trajectory and by the mask after Cartesian resampling. A numerical phantom was created to assess the fidelity of the images and time courses generated using GST-MRA. The simulated dataset was made up of 2000 projections with a base resolution of 2562 and different time courses to mimic arterial and venous flow. In order to combine coil sensitivity

information into the reconstruction, each time frame image was multiplied by a simulated 8-channel receiver coil. The composite image was generated using all 2000 projections for each of the 8 channels, and the coil sensitivity map was generated using adaptively combined images [8] and the composite image. GST-MRA reconstructions were performed using 15 projections per time frame (R=20) and a composite threshold level of 10% of the maximum pixel value. In order to examine in vivo GST-MRA reconstructions, CE-MRA data were acquired on a patient with an arterial-venous malformation (AVM) with the following parameters: TR=3ms, TE=1.5ms, total projections=1344, 75% asymmetric echo, Matrix=192x192, FOV=220x220 FA=20°, Partitions=15, slice thickness=4mm. The composite image was generated using all of the collected projections, and GST-MRA reconstructions were generated using 15 projections per frame and a threshold level of 10%. Results: The arterial and venous time courses for the GST-MRA reconstruction of the numerical phantom are shown in Figure 2. GST-MRA accurately follows both with limited streaking in time

structed using GST-MRA are shown in the bottom row of Figure 3, with the corresponding gridded images shown in the top row. Representative time courses of arterial and venous pixels are shown in Figure 4 for both reconstruction methods. GST-MRA clearly improves the spatial localization and removes streaking artifacts and venous contamination as compared with NUFFT.

Discussion: GST-MRA is a method for reconstructing sparse MR images with highly undersampled data using the Generalized Sampling Theorem in conjunction with a composite mask and coil sensitivity maps. This method improves the reconstruction fidelity, especially in later time frames, where there is more potential for spray from other vessels. One potential drawback to the method is that a threshold must be determined empirically; if this threshold is set improperly, either small vessels are masked out, or streak artifacts remain in the final image. In these reconstructions, however, the threshold choice has not been critical, and values between 10% and 15% offer similar image quality. In implementation, GST-MRA resembles both I-HYPR [3] and CG-HYPR [4]; the difference between them is that GST-MRA employs a masking step including coil sensitivity maps, further constraining the final reconstructed image. As

evidenced by the simulation results and in vivo images, GST-MRA offers highly temporally resolved images with few streak artifacts and little venous contamination when using very small numbers of projections (15 for a base matrix of 192² in vivo). Thus, GST-MRA is promising for applications where both high spatial and temporal resolution are required, such as MR angiography.

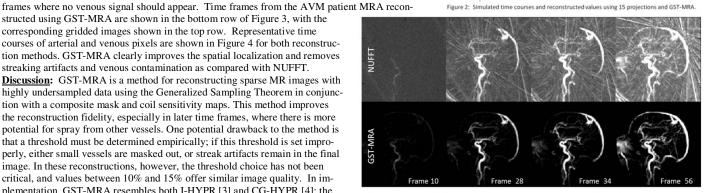


Figure 3: NUFFT gridded images (top) and GST-MRA reconstructions (bottom) for several frames of an in vivo CE-MRA acquisition.

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NUFFT Arterial Signal

Figure 1: Schematic of GST-MRA

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gure 4: Representative time courses for the in vivo NUFFT and GST-MRA reconstructions

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