Investigation of Sparsifying Transforms for Compressed Sensing in MRI Reconstruction

C. Baker[1], K. King[2], D. Liang[1], and L. Ying[1]

[1]Eletrical Engineering, University of Wisconsin at Milwaukee, Milwaukee, WI, United States, [2]Global Applied Science Lab, GE Healthcare, Waukesha, WI, United States

Introduction

Compressed Sensing (CS) image reconstruction requires the image of interest to be sparse or compressible under some transform and the encoding matrix to be incoherent with the sparsifying transform. With Fourier encoding in MRI, the level of sparsity and incoherence achieved by the transform affects the under-sampling that can be performed (1,2). The most desirable transforms achieve a high level of compressibility for the images to be reconstructed and a high level of incoherence for Fourier encoding. Wavelets have been proven to be incoherent with the Fourier basis and also shown to be a good sparsifying transform (2,3). This work investigates contourlets (4) and the discrete cosine transform (DCT) as transforms for CS MRI reconstruction and compares them with the widely used wavelet transform.

Methods

Both the contourlet, a very special non-orthogonal transform, and the classical DCT are known to be effective in image compression. To evaluate their effectiveness in CS, both transforms were used as the sparsifying transforms for CS reconstruction. Specifically, the image vector *m* was solved for by minimizing $\|Wm\|_1$ under the constraint $Em = y$, where E is the Fourier (in the case of single-channel) or sensitivity (in the case of multi-channel) encoding matrix, y is the reduced k-space data, and W is the chosen sparsifying transform. The reduced k-space data were obtained by pseudo-randomly under-sampling the fully acquired phase encodings with variable density according to the CS principle (3). The constrained minimization was performed using a nonlinear conjugate gradient algorithm (5). For the contourlet as a sparsifying transform, a slightly over-complete basis was used to transform the entire image, producing at most $4/3^{rds}$ the number of coefficients as the image size. The DCT used can have an over-complete basis and is used to transform small and overlapping patches of the image in a way similar to references 6 and 7. An Orthogonal Matching Pursuit (OMP) algorithm is used to compute the coefficients θ for the image m with the over-complete DCT basis Ψ by minimizing $\|\theta\|_0$ under the constraint $\Psi\,\theta = m$ (7). CS reconstruction with the wavelet transform as the sparsifying transform was also performed on the image as a whole. An in-depth search and comparison of a vast number of different settings such as filters and decomposition levels were performed to identify good parameters to use for each data set and testing scenario. All reconstructed images were evaluated by contrasting with the fully sampled reconstructions. Quantitative and qualitative analyses were performed by observing changes in feature structure, artifacts, noise, and mean-squared-error (MSE). Additionally, observations of different algorithm parameters such as, number of iterations, and gradient calculation approaches were also considered. Several image datasets were tested with different features and contrast, to determine whether contourlets or DCT on patches could outperform wavelets for some types of MRI. To make a fair comparison, no total variation (TV) was included in the minimization, although it can improve the image quality.

Results

The CS reconstruction results with different transforms were compared by visually inspecting the images after a proper number of iterations of the non-linear CG algorithm. An eight-channel dataset and a single channel dataset of a brain with T1-weighting were chosen as the test data. A reduction factor of two was used in the single channel dataset and a reduction factor of 3.3 was used for the eight-channel dataset. Figure 1 and 2 show the reconstructed images. CS with the contourlet transform performs similarly to CS with the wavelet transform; the visual comparisons between the two reconstructed images show very minimal differences in structure, artifact suppression and noise. In contrast, CS with the DCT on patches was able to remove more artifacts caused by the under-sampling and reduce more noise across the image, while still preserving the image details. Quantitatively, the DCT on patches had a 15-25% improvement in MSE measurement compared with the wavelet and contourlet transforms.

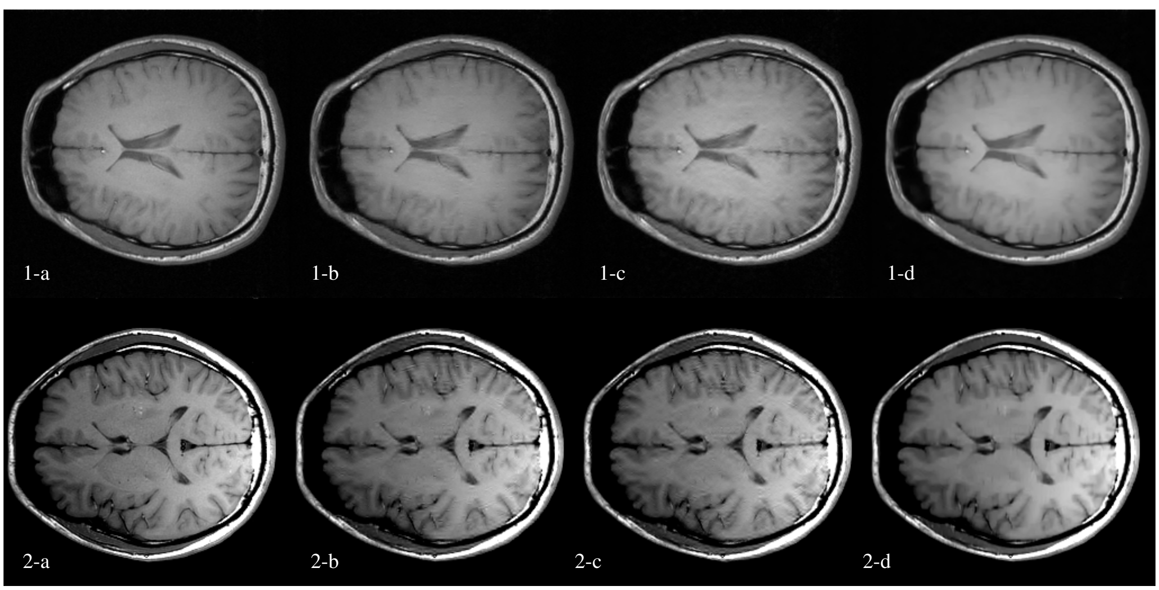

Fig 1: Single Channel CS reconstruction with reduction factor of 2 for b through e, (a) Fully sampled, (b) with wavelets, MSE = 0.0041, (c) with contourlets, MSE = 0.0048, (d) with DCT on patches, MSE = 0.0033, Fig 2: Multi-channel CS reconstruction with reduction factor of 3.3 for b through e, (a) fully sampled, (b) with wavelets, MSE = 0.0054, (c) with contourlets, MSE = 0.0058, (d) with DCT on patches, MSE = 0.0046

Conclusion

The contourlet transform and the DCT on patches were investigated as sparsifying transforms for CS reconstruction. Results show that the contourlet transform performs about the same as the wavelet, while the DCT on patches outperforms the wavelet in CS reconstruction of MR images. The observation suggests that use of DCT on small image patches may improve the CS reconstruction quality.

References

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