Automatic regularization for magnetic resonance inverse imaging

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INTRODUCTION

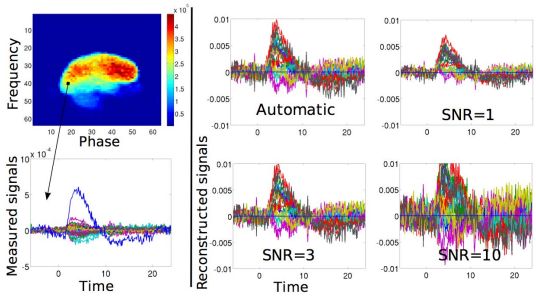
Magnetic resonance Inverse Imaging (InI) [1] is inspired by magnetoencephalography (MEG) and electroencephalography (EEG), where the localization of neural activity (being based on the extra-cranial measurements) necessitates a solution to an ill-posed inverse problem [2]. The ill-posedness means that the solution is not unique and/or can be sensitive to the measurement noise, and the problem needs regularization. In inverse imaging, a parallel radio-frequency coil array is used to obtain spatial information and to reduce the acquisition time by omitting some of the gradient encodings. Unlike in MEG and EEG, the degree of ill-posedness of the inverse problem in InI can be varied by considering different spatial encoding schemes. We propose a method for making the regularization procedure automatic for inverse image reconstructions by using Bayesian statistical inference [1,3].

METHODS

The image reconstruction in InI corresponds to estimation of a linear regression model $\mathbf{Y}(t)=\mathbf{A}\mathbf{X}(t)+\mathbf{n}$, $t=1,\ldots,T$. Such a linear model is solved independently for each gradient-encoded spatial location. Here we adopt the phase constraint, so that the vector $\mathbf{Y}(t)$ represents the stacked real and imaginary parts of the complex signals of the receiver coils. The matrix $\mathbf{A}$ is obtained by performing a full spatial-encoding reference scan, $\mathbf{X}(t)$ is the vector of voxel intensities to be estimated, and the noise $\mathbf{n}$ is assumed Gaussian with zero mean and a time-independent covariance $\mathbf{C}$. The ill-posedness manifests itself in the matrix $\mathbf{A}$, which can have a nontrivial null-space (more unknowns than equations), and/or small singular values (sensitivity to noise). In the ill-posed case, the Maximum Likelihood (ML) estimate for $\mathbf{X}(t)$ does not exist in a meaningful sense. In the Bayesian approach the situation is remedied by assuming a zero-mean Gaussian prior for the $\mathbf{X}(t)$ with a covariance $\gamma\mathbf{1}$. If the regularizing prior variance γ is known, the inverse solution can be obtained as the classical (regularized) Minimum-Norm Estimate (MNE): $\mathbf{X}_{MNE}(t) = \gamma\mathbf{A}^T(\gamma\mathbf{A}\,\mathbf{A}^T+\mathbf{C})^{-1}\,\mathbf{Y}(t)$, which corresponds to the Maximum A Posteriori (MAP) estimate under the modeling assumptions. Typically, the parameter γ is not known, and must be set according to some more or less rigorous principles. In [4], the regularization was set by the *ad hoc* criterion $\lambda:=1/\gamma=\{\text{trace}(\mathbf{C})/\text{trace}(\mathbf{A}\,\mathbf{A}^T)\}/\text{SNR}$. The parameter SNR is a pre-defined quantity reflecting the overall signal-to-noise ratio of the data. Proceeding in a mathematically more rigorous way, it is well-known that given γ the joint probability density $p(\mathbf{Y}(t),\mathbf{X}(t)|\gamma)$ is Gaussian. As the noise is independent of time, we can multiply the probabilities of consecutive time-points and integrate over the unknown $\mathbf{X}(t)$'s in a closed form, resulting in $p(\mathbf{Y}(1),\ldots,\mathbf{Y}(T)|\gamma)$. This is called the marginal likelihood of γ, and we can obtain a (type-II) ML estimate for the regularization parameter by setting $\gamma_{ML}=\text{argmax}\; p(\mathbf{Y}(1),..,\mathbf{Y}(T)|\gamma)$. There are many methods for optimizing the marginal likelihood such as the EM algorithm, but here we simply evaluate this function over a discretized domain, and obtain γ_{ML} as the argument corresponding to the largest marginal likelihood value. After γ_{ML} is estimated in this automatic fashion, the MNE is again trivial to compute. We tested the method with a visual dataset previously studied in [4], where a 32-channel InI setup was used to reconstruct a 3D (64x64x64) volume from 2D (64x64) gradient encoded signals at 100 ms sampling rate.

RESULTS

The upper left panel of the figure shows the values of λ calculated by using the *ad hoc* method and SNR=1 for the 2D gradient encoded locations. For evaluation of the regularization methods, we selected one location corresponding roughly to visual cortex for which the receiver coil time-courses $\mathbf{Y}(t)$ are shown in the lower left panel. In the right panel, we illustrate four cases for InI reconstructions $\mathbf{X}_{MNE}(t)$, where the γ_{ML} was used in the automatic method, and three SNR parameter values were assumed for the *ad hoc* regularization. In comparison to the maximum marginal likelihood, the SNR=1 assumes too much regularization, and the SNR=10 too little. The maximum marginal likelihood estimate $\gamma_{ML}=1.12 \times 10^{-5}$ corresponds to SNR=3. The present results are in line with those obtained in [4].



DISCUSSION

Regularization plays a significant role in many aspects of parallel acquisition MRI, and here we presented a simple automatic method utilizing marginal likelihood. As clearly seen in the above figure, for dynamic MRI the regularization should actually be time-dependent, as the baseline measurements consist mainly of noise. The presented method can be extended to the time-dependent case, but a dynamical model must be incorporated, since our preliminary results indicate that individual type-II ML estimates for the $\gamma(t)$ are not necessarily robust against noise.

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