Spline-based variational reconstruction of variable density spiral k-space data with automatic parameter adjustment

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Introduction:

Small-animal cardiac imaging shows a renewed interest, but spatial and temporal resolution are still a challenging problem as well as movement and flux artifacts that alter considerably the image quality with traditional acquisition techniques. One possible way to assess those problems is the use of non-Cartesian acquisition scheme like variable density spiral (VDS) [1]. The main problem lies in image reconstruction that cannot be done by inverse Fast Fourier Transform (FFT). In this context, the regridding algorithm [2],[3] owes its popularity as it transforms the data onto the classical Cartesian grid, after which the efficient FFT inversion can be applied. However, regridding introduces noticeable artifacts due to *k*-space interpolation, especially when dealing with undersampled trajectories. Here, we propose a variational approach where the image is described by a spline model [4]. Moreover, an automatic adjustment of the regularizing weight is implemented based on the principle of crossvalidation. We evaluate our framework for various degrees of the spline model and different orders of derivation of the regularizer.

Methods:

We use the following MRI measurement model:

$$s(k_x,k_y)=\int \rho(x,y)e^{i2\pi(k_x x+k_y y)}dxdy+n(k_x,k_y) \quad (1) \quad \text{with} \quad \rho(x,y)=\sum_{k,l=0}^{N-1}c[k,l]\beta^{\alpha}(x-k)\beta^{\alpha}(y-l),$$

where the image ρ is described by its spline coefficients $c[k,l]$ on a conventional $N\times N$ grid, and where $n(k_x,k_y)$ represents the (complex-valued) noise. This formulation does not impose the (usual) band-limitedness of the image, which can lead to Gibbs ringing artifacts in the reconstruction. As the direct inversion of (1) is ill-posed for undersampled trajectories, we adopt a variational approach; i.e., we minimize the functional $J(\rho) = \|s - H\rho\|^2 + \mu R\{\rho\}$ (2), which consists of a dataterm that acts on the measurements (l_2-norm) and the regularizer $R\{\rho\}$ that operates on the continuous-domain description of the image ρ (L_2-norm). For noisy measurements on undersampled trajectories, regularization adds prior information that makes the problem well conditioned. Here, we have chosen the regularizer $R\{\rho\} = \|D_x^{\gamma}\{\rho\}\|^2 + \|D_y^{\gamma}\{\rho\}\|^2$ where γ is the order of derivative; e.g., γ=1 corresponds to classical Tikhonov regularization. Due to the spline model, we can access continuous-domain derivatives of ρ, for order $\gamma \leq$ spline degree α, by (efficient) discrete filtering. To effectively minimize (2), a gradient descent algorithm computes $\partial J/\partial c$ essentially as a shift-invariant convolution where the kernel can be precomputed (equivalent to $H^T H$); this method can be seen as a spline generalization of the gradient expressions by Wajer [5] and Boubertakh [6]. As the convolution kernel is large for undersampled non-Cartesian trajectories, it can best be implemented using FFTs and products. The regularization weight μ plays an important role in the reconstruction quality. An automatic method sets this parameter by evaluating the data term on a validation set of *k*-space data; i.e., one out of the *M* interleaves is used in an alternating way to increase or decrease μ, see Figure 1.

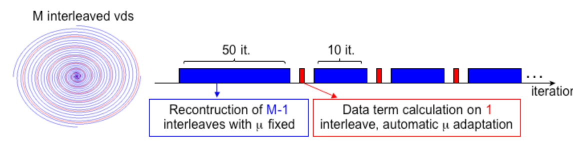

Fig. 1 : Automatic setting of the regularization weight μ.

Results and discussion:

We have validated our reconstruction method on the Shepp-Logan synthetic phantom (using its analytical form in *k*-space). The trajectory used a VDS composed of 21 to 26 interleaves, the fixed parameter was the number of turns of each interleave (i.e. nt=4, minimum to keep properties of VDS) in order to have the same TR. The proposed reconstruction method was compared to classical regridding with Kaiser-Bessel kernel. Table 1 summarizes the SNR values obtained for trajectories that were sampled at different densities with respect to the Nyquist criterion (i.e. at the edge of k-space, the space between the two last VDS turns Δk is evaluated in order to respect Nyquist criterion Δk=1/FOV, or is reduced to 90% or 67%), for different noise levels, for different degrees α of the spline model, and for different regularization order γ. In all cases, the SNR of our method largely outperforms the one of regridding. For high sampling density (100-90%), the cubic (α=3) spline model with first-derivative regularization (γ=1) works best for both noiseless and noisy measurements (complex random noise was added on k-space to reach a SNR=21 dB). For lower sampling density (67%), the band-limited assumption is more violated, which makes the spline model of lower degree more appropriate (α=1). Most notably, the well-known background artifacts found in regridding results (see Fig. 2) are eliminated. In Fig. 3, we show the feasibility of our method for experimental data acquired using a spiral trajectory where only the half of k–space was sampled (i.e. $kmax_{acqu}$ = kmax/2) (Siemens Tim Trio 3T, TA/TE 100/3.6 ms, resolution 3.8x3.8x2.5 mm^3). Our reconstruction reveals better contrast homogeneity compared to regridding.

		Noiseless			Noise $SNR_{k\text{-space}}$=21dB		
α	D^{γ}	Full	90%	67%	Full	90%	67%
0	0	10.59	10.05	7.32	6.99	6.29	7.95
1	0	9.41	9.34	6.75	5.88	5.46	7.26
	1	10.88	9.56	**9.71**	6.79	6.32	**8.47**
3	0	11.44	10.79	5.99	7.88	7.58	6.32
	1	**11.97**	**10.88**	6.67	**9.59**	**8.44**	6.95
	2	10.85	10.02	6.57	8.67	7.86	6.96
regridding		**5.10**	**4.66**	**4.69**	**3.30**	**3.04**	**1.72**

Table 1 : SNR for different α, γ, and sampling density. The noisy case represents 21 dB as measured on the k-space samples.

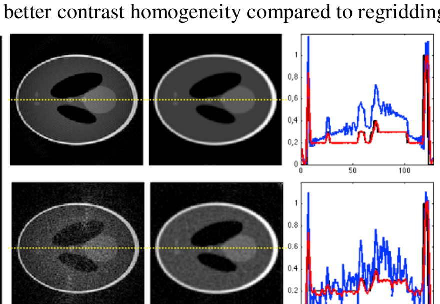

Fig. 2 : (Up, left to right) : Regridding for noiseless 100%; Spline-based reconstruction for α=3, β=1; Horizontal profiles at center with original (black), regridding (blue), spline-based reconstruction (red, essentially matches the original black). (Down, left to right) **:** Regridding for noisy 67%; Spline-based reconstruction for α=1, β=1; Horizontal profiles.

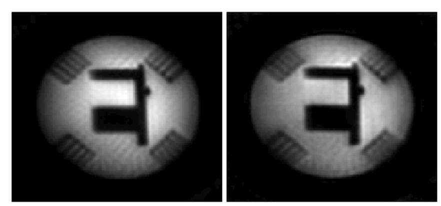

Fig. 3 : Regridding (left) ; Spline-based reconstruction (right) for real data using spiral trajectory. We observe better contrast homogeneity in spline-based reconstruction.

Conclusion:

The proposed spline-based variational approach offers great flexibility for reconstruction of VDS trajectories. Robustness and reproducibility where obtained by using the automatic setting of the regularization weight. We evaluated the reconstruction quality on a synthetic dataset for which SNR values and intensity profiles show excellent performance as compared to classical regridding. While we investigated linear regularizers until now, the proposed methodology can be easily extended to deal with non-linear regularization such as total variation or sparsity (e.g., compressed sensing [7]).

References : [1] Kim et al. MRM (2003) 50(1): 214-9 ; [2] O'Sullivan IEEE Trans Med Imag (1985) MI4:200-207 ; [3] Jackson et al. IEEE Trans Med Imag (1991) 10:472-478 ; [4] Unser M. Signal Processing magazine IEEE (1999) 16(6):22-38 ; [5] Wajer et al. Proc. ISMRM 9 (2001) ; [6] Boubertakh R et al. Signal Processing (2006) 86(9):2479-2494 ; [7] Lustig et al. MRM (2007) 58:1182-1195