In-vivo-Sensitivity-Based Regularization of Parallel MR Image Reconstruction

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Introduction

One requirement that different parallel imaging techniques have in common is the need to determine spatial sensitivity information for distinct coil array elements. In the SENSitivity Encoding (SENSE) technique, this is usually done by postprocessing low-resolution calibration images, e.g. via polynomial fitting [1]. This postprocessing introduces a possible source of error into the reconstruction. Another important factor in parallel imaging is noise amplification in the reconstruction, which results from poor conditioning of the encoding matrix inversion produced by the spatially correlated coil sensitivity profiles. Generally, regularization approaches, such as Tikhonov [2] and SVD-based methods, are applied to reduce SNR loss, at the price of introducing residual aliasing. In this work, we present a regularization approach using raw images sometimes referred to as "in vivo coil sensitivities". We demonstrate that this approach can removes noise and preserve image integrity more efficiently that regularization with postprocessed sensitivities.

Method

Cartesian SENSE acquisition can be modeled as a matrix multiplication $|S\rangle = C|X\rangle$, where $|S\rangle$ is a vector representing the aliased component coil image set, $|X\rangle$ is

the target unfolded image, and C is an encoding matrix composed of complex coil sensitivities at each set of aliased positions in the target image. The most common solution to estimation of the coil sensitivities that populate the matrix C has been to perform some form of smoothing operation on low-resolution acquired calibration images. This postprocessing step can introduce error into the parallel image reconstruction, particularly in low-signal anatomical regions such as the lung. SENSE reconstruction can be reduced to a simple matrix inversion problem $|X\rangle = C_{inv}|S\rangle$, which can be generally solved by pseudo-inverse or singular value decomposition

(SVD). At high acceleration factors, the encoding matrix C tends to be dominated by one or more small singular values, and its inversion may result in large noise amplification factor (or g-factor) in the reconstructed images. Several regularization approaches have been proposed to limit noise amplification. To simultaneously address coil sensitivity calibration and regularization of the reconstruction, we propose the use of *in vivo* coil sensitivities [3] combined with an SVD-based

regularization approach to achieve reconstructions that are robust to noise and residual aliasing in comparison to traditional approaches. "In vivo coil sensitivities" refers to the direct use of low resolution anatomical images as the coil sensitivities for reconstruction. In this case, the nominal *in vivo* encoding matrix \hat{c} is actually a multiplication of the

underlying signal and the actual coil sensitivity at each pixel, i.e. $\widehat{C} = C\rho$. In other words, the true coil sensitivity is modulated by the reference image. In self-calibrating parallel imaging techniques, this low-resolution reference image is obtained from a small portion of the center of k-space sampled during an otherwise accelerated acquisition. Using this new

coil sensitivity matrix to perform SENSE reconstruction $|\widehat{x}\rangle = \widehat{C}_{sw} |S\rangle = \rho^{-1}C_{sw} |S\rangle = \rho^{-1}|X\rangle$. In this case, \widehat{C}_{sw} contains information both from the coil sensitivity and from the underlying magnetization density. As a result, after SVD, the signal is more concentrated into a few singular vectors than it is during a standard reconstruction with pure coil sensitivities. Fig.1 illustrates this point via a mathematical simulation. A SENSE acquisition with acceleration factor of 4 was simulated with a T₁ weighted brain image. The sub-image associated with each singular value/vector k is shown. Both a traditional coil-sensitivity based approach and



(a) (b) Fig. 1: Sub-images reconstructed from each singular value/singular vector by using (a) *in vivo* coil sensitivities and (b) pure coil sensitivities.

the proposed *in vivo* sensitivity based approach were tested. The results of Fig.1 verify the fact that using *in vivo* sensitivities will result in better separation of signal and noise into different channels of the SVD than a pure coil-sensitivity based approach. This property reduces the residual aliasing artifact when applying regularization. For regularization, we used a shifted-SVD (SSVD) method [4]. Shifted-SVD regularization shifts the spectrum of singular values away from zero by adding a small portion of the largest singular value, which significantly reduces the effect of small singular values (noise) in the inverse while maintaining the effect of large singular values (signal). In the bra-ket formulation used here, SSVD can be expressed as: $C_{m_{-}SSVD} = VS^{-1}U^{+} = \sum_{k} |v_{k}\rangle (s_{k} + \delta)^{-1} \langle u_{k}|$, where δ is a small portion of the largest singular value.

Results

The feasibility of our proposed method was demonstrated in a whole-heart breath-held 3D acquisition with a 4x2-fold two-dimensional acceleration using a 32-element coil array. A separate fully-sampled low resolution scan was performed for generation of in vivo coil sensitivities. Fig. 2 shows one slice of the whole-heart 3D reconstruction using SENSE with in vivo coil sensitivities. The shifted-SVD regularization method substantially reduces noise in low-signal regions of the image without introducing visible residual aliasing artifacts.

Conclusions

This paper presents preliminary results and the technical feasibility of demonstrates regularization of Parallel MR image reconstruction using in vivo coil sensitivities. This approach not only combines coil sensitivity calibration with regularization of the inverse problem, but also provides better performance than a traditional two-step scenario. Ultimately, this approach could be applied to any SENSEbased parallel image reconstruction, especially at high acceleration factors and/or when pure coil sensitivity profiles are difficult to estimate.



Fig. 2: Representative axial slice from a SENSE-reconstructed 3D whole-heart acquisition with 4x2 acceleration using in-vivo coil sensitivities. (a) no regularization and (b) shifted-SVD regularization with a shift of 10% of the maximum singular value.

References

[1] K. Pruessmann, et al, MRM 1999, 42:952-962. [2] D. Sodickson, MRM 2000, 44 :243-251. [3] R. Otazo, et al, MRM 2007, 58 :1107-1116.