SVD eigenimage based SENSE

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Introduction: In this study, an eigenimage theory based on singular value decomposition (SVD) is proposed for SENSE [1]. Using SVD, a set of eigenimages can be generated from the reduced FOV images and the SENSE image can be represented as the linear combination of these eigenimages. As a result, the matrix inversion in SENSE is treated as the calculation of a set of linear coefficients for weighting the eigenimages. Based on this theory, a group of techniques for SENSE optimization has been developed. As an example, this abstract presents a data-driven SENSE regularization technique, where the regularization parameters are generated from the SNR of acquired data. This technique has advantages over several previously proposed regularized SENSE techniques, where the regularization parameters have to be determined either from experience [2] or from *a priori* information [3]. Moreover, our experiments demonstrated that this technique can suppress not only the noise amplification, but also the artifacts introduced by sensitivity map calibration.

Theory: Consider a set of reduced FOV data acquired from an N-channel RF coil array with a reduction factor of R. The standard SENSE unfolding matrix $(\mathbf{S}^H\mathbf{\Psi}^{-1}\mathbf{S})^{-1}\mathbf{S}^H\mathbf{\Psi}^{-1}$ [1] can be written as *pinv*($\mathbf{\Phi S}$), where "*pinv*" represents the pseudo-inverse, the matrix $\mathbf{\Phi}$ is generated from the noise correlation matrix $\mathbf{\Psi}$ by $\mathbf{\Psi}^{-1}=\mathbf{\Phi}^H\mathbf{\Phi}$ and $\mathbf{S}$ is the sensitivity matrix. By applying SVD, $\mathbf{\Phi S}$ can be represented by Eq. 1, where d_i's are eigenvalues, and $\mathbf{u}_i$'s and $\mathbf{v}_i$'s are the corresponding eigenvectors for the matrices $\mathbf{\Phi SS}^H\mathbf{\Phi}^H$ and $\mathbf{S}^H\mathbf{\Phi}^H\mathbf{\Phi S}$ respectively. From SVD, it is easy to obtain Eq. 2, which gives a different approach to calculate the SENSE unfolding matrix. In addition, the new concept of eigenimages is defined in Eq. 3, where E_i is the ith eigenimage ordered by magnitude of eigenvalues, and $\mathbf{a}$ is the reduced FOV image (or aliased image). As a result, the final SENSE image is a weighted summation of eigenimages (Eq. 4). Using this new eigenimage concept, it can be seen that the accuracy of SENSE reconstruction depends on the estimate of weighting coefficients k_i's and the SNR of eigenimages. It is therefore possible to optimize the total SNR in SENSE by modifying the coefficients for weighting the eigenimages according to their individual SNRs. This leads to Eq. 5, which describes the key idea in the data-driven regularized SENSE reconstruction. It is important to mention that in the definition according to Eq.5, λ is a normalization parameter and has the same value for all pixels and for all eigenimages. For the special case when λ=0 in Eq. 5, the Eq. 4 will be reduced to the original SENSE formulation by Pruessmann. In our current implementation we chose λ to be defined according to Eq. 6. This choice appears to be a good compromise between maximizing SNR and minimizing residual artifact.

$$\mathbf{\Phi S} = \sum_{i=1}^{R} d_i \mathbf{u}_i \mathbf{v}_i^H \qquad (1)$$

$$pinv(\mathbf{\Phi S}) = \sum_{i=1}^{R} d_i^{-1} \mathbf{v}_i \mathbf{u}_i^H \qquad (2)$$

$$E_i = \mathbf{v}_i \mathbf{u}_i^H \mathbf{a}; \qquad i = 1, 2, \ldots, R \qquad (3)$$

$$\text{SENSE Image} = \sum_{i=1}^{R} k_i E_i \qquad (4)$$

$$k_i = \frac{1}{\left(d_i + \lambda / SNR_i\right)}; \qquad i = 1, 2 \ldots, R \qquad (5)$$

$$\lambda = 0.002\,mean(d_1 E_1) \qquad (6)$$

Methods: An *in vivo* experiment was performed using an 8-channel head coil (Invivo, Gainesville, FL) on a 3.0T Achieva scanner (Philips, Best, The Netherlands). A set of transverse brain images was acquired with full Fourier encoding using a spin-echo sequence (matrix 256×256, FOV 230 mm, TR 2000 ms, TE 15 ms, flip angle 90°, slice thickness 4mm, NSA 1). Phase encoding direction was left-right. The data was artificially under-sampled with R = 4 to simulate the reduced FOV acquisition. Sensitivity maps were calculated using the low-resolution images generated from 24 central k-space encoding lines. The pixel-wise SNR in the eigenimages were calculated by normalizing our weighting factors $\mathbf{v}_i\mathbf{u}_i^H$ in such a way that the combined noise level = 1 [4]. This operation requires knowledge of the actual noise correlation matrix $\mathbf{\Psi}$ for the original data.

Results: Fig. 1 shows reconstruction results using the standard SENSE reconstruction. It can be seen that the g-factor is not only affected by singularities in the matrix inversion but also by ringing artifacts due to the use of low-resolution data for the sensitivity maps. As a result, the reconstruction error is significant. The use of eigenimages offers the possibility to find the major sources for noise and artifacts. Fig. 2 a-d shows the four eigenimages generated using Eqs. 1-3. It can be seen that the 4th eigenimage mainly contains artifacts and noise and its SNR is very low. Using Eq. 5, it is easy to suppress the weight for the 4th eigenimage in the SENSE image reconstruction in Eq. 4. Fig. 3 shows the reconstruction results. It can be seen that g-factor and reconstruction error have been significantly reduced.

Discussion: The experimental results demonstrate the effectiveness of eigenimage concept and the eigenimage-based regularization technique proposed in Eq. 5. The eigenimages defined in Eq. 3 mathematically describe modes corresponding to k-space undersampling pattern. Because the artifact and noise level is different in the different mode, the use of eigenimages offers an efficient way to control the suppression of artifact and noise in SENSE. Based on eigenimage theory, a group of SENSE optimization techniques has been developed. Currently, these techniques are being investigated.

Reference: 1). Prussmann, K.P. et. al., MRM 42: 952-962 (1999). 2) King, K. F. et. al ISMRM 2001, p1771. 3) Tsao J, et al. Proc. ISMRM. 2002, p739. 4). Roemer, P.B., et. al., MRM 16: 192-225 (1990).

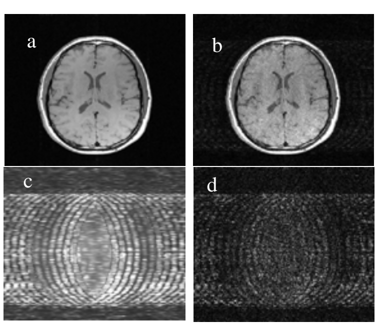

Fig. 1. Standard SENSE reconstruction (a) Reference image from fully-sampled data; (b) SENSE image; (c) g-factor map; (d) Difference map between (a) and (b).

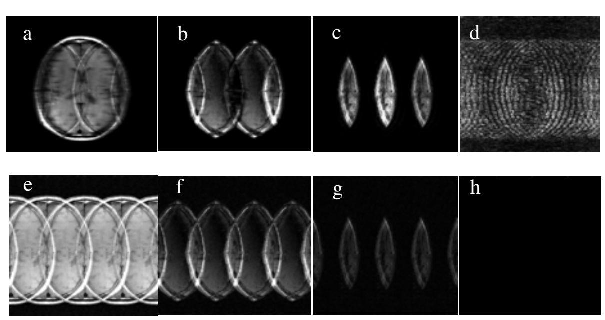

Fig. 2 (a)-(d): Eigenimages generated from Eqs. 1-3. (e)-(h): Corresponding SNR maps.

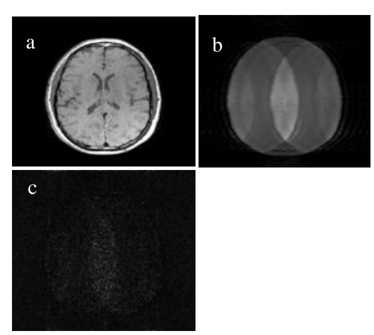

Fig. 3 Reconstruction using SENSE with the eigenimage-based regularization. (a) Reconstructed image; (b) g-factor map; (c) Difference map between (a) and Fig. 1(a).