A Low Curvature Uniformly Sampled Trajectory for 3D Imaging: The Interlocking Loops Trajectory

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Introduction

For fast volumetric imaging, many trajectories have been introduced to sample 3D k-space efficiently. For maximal time efficiency, two key properties are uniform sampling, and low curvature. The first minimizes redundancy of data, and the second reduced requirements of gradient slew rate, thereby allowing for maximal gradient amplitude and k-space velocity. To our knowledge, no existing trajectories provide both uniform sampling and uniformly low curvature. Center-out trajectories such as projection reconstruction, cones, and their derivatives are oversampled at the center of k-space. More uniform trajectories such as echo-volumar imaging and stack-of-spirals have high curvature at the u-turns, and along the axis of the spirals, respectively. We introduce here a trajectory that uniformly samples a sphere, with low curvature at all points.

Methods

The trajectory consists of two interlocking bundles of loops. The trajectories through each bundle were derived numerically using simple interlocking toroidal rings as a staring point, and applying at each iteration the following forces and constraints:

- 1. All points are constrained to lie within a unit sphere
- 2. Trajectories repel one another using a $1/r^n$ force law, with n ranging from 1.5 to 2.
- 3. Curvature is reduced by local smoothing using a second order local smoothing function.
- 4. The two bundles are constrained to be identical.

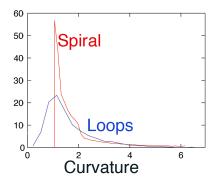
For the example shown below, 51 loops within each of the two bundles are described by 45 points each. However, the overall shape of the bundles is nearly independent of the number of loops. Within the cross section of the bundle, the initial distribution of loops was described by a Fibonacci spiral for maximum randomness.

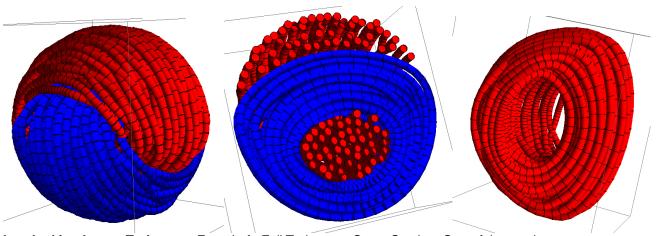
Results

The loops trajectory is shown in the Figures below. The distribution of curvature (inverse of radius of curvature) is shown in the plot on the right. For comparison, the curvature of a stack-of-spiral trajectory is also shown (in red). Note that the minimum curvature of the spiral trajectory is 1 (at the perimeter of the spiral), while that of the loops trajectory is lower. The mean and peak curvatures of the loops trajectory is also lower than those of the spiral.

Discussion

The loops trajectory introduced here has lower peak and mean curvature than other uniformly sampling 3D trajectories. This allows for higher average k-space velocity, and thereby faster scan times.





Interlocking Loops Trajectory. From Left: Full Trajectory; Cross Section; One of the two loops.