Uncertainty in VFA T1 mapping with multiple flip angles

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Introduction: We have used propagation of errors, in conjunction with the theoretical signal equation for a spoiled gradient echo pulse sequence, to derive a closed-form theoretical expression for uncertainty in T1 mapping using the variable flip angle (VFA) method with two flip angles (1).

$$\min \epsilon = \frac{1}{\mathrm{SNR}} \frac{2T_1\sqrt{(1-E_1)(E_1+1)^3}}{T_R E_1 E_2} \qquad (1)$$

Minimization of this expression results in a solution for the optimal flip angles that elucidates a well-known empirical result (2). Combining these two results, we find the simple expression representing a lower bound on the achievable T1 measurement uncertainty for a given set of pulse sequence parameters and signal-to-noise ratio (SNR) given in Equation 1. Monte Carlo simulations for VFA measurements with more than two flip angles strongly suggest that the theoretical lower bound is equally applicable to multi-angle measurements as well, consistent with arguments in (3).

Methods: Theoretical expressions for the two angle case were derived using conventional methods of error analysis. Monte Carlo simulations were performed for TRs of 5 ms and 800 ms, with TE = 2 ms, T2* = 50 ms. SNR was set to 500 for TR=5 ms and 39.5 for TR=800 ms to maintain constant noise efficiency. One set of simulations consisted of 4000 realizations for each repetition time for N=2 to N=10 flip angles, with the N flip angles drawn from a uniform random distribution spanning the range of optimal flip angles for a two-flip angle measurement for each T1 value. Gaussian-distributed noise was added to each simulated measurement and 10000 noise realizations were computed for each of the 40 T1 values (50-3950 ms) in the joint PDF. A second set of simulations was performed in the same way, but with N = 2-50 flip angles uniformly distributed between the optimal flip angles and 1000000 noise realizations for each T1 value. For all simulations, standard linear regression was used to estimate T1.

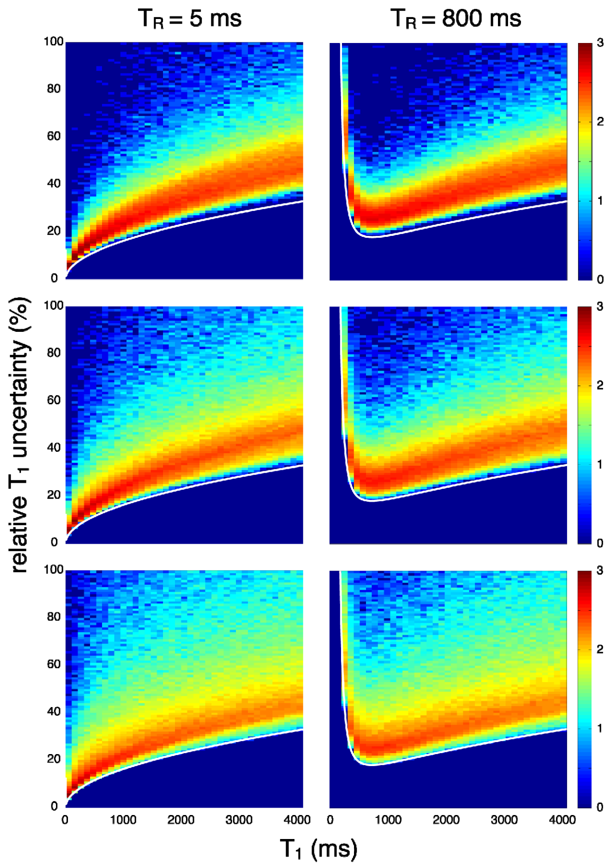

Figure 1: Logarithmic joint PDFs of T1 uncertainty as a function of T1 for N=4 (bottom row), N=6 (middle row), and N=10 (top row) flip angle VFA measurements. The theoretical minimum error bound given by Eq. 1 is indicated by the solid white curve.

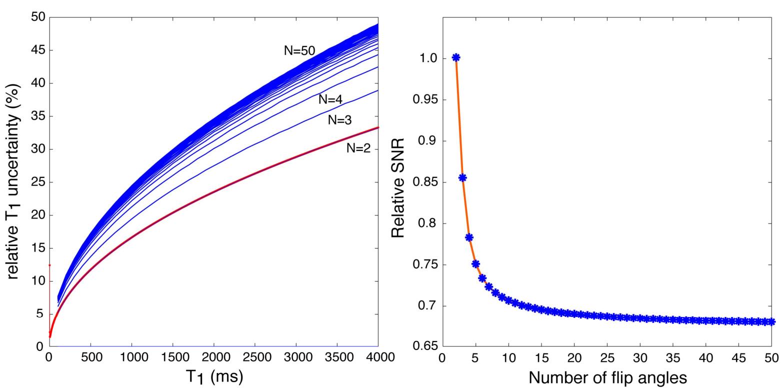

Figure 2: Multiple flip angle VFA minimum relative T1 uncertainty as a function of T1 for N=2-50 for uniformly distributed flip angles (left panel). Relative SNR at constant acquisition time vs. N is shown in the right panel.

Results: Figure 1 plots the joint PDFs of relative T1 uncertainty estimated from the Monte Carlo simulations with randomly distributed flip angles for N=4 (bottom row), N=6 (middle row), and N=10 (top row) for TR=5 ms (left column) and TR=800 ms (right column). Figure 2 plots simulated T1 uncertainty for VFA measurements made with N evenly-spaced flip angles, where N ranges from 2 to 50 (left) and the corresponding relative SNR (right).

Discussion: From Figure 1 it is clear that the average performance for N>2 is significantly worse than for the two angle case when flip angles are randomly chosen. In addition, none of the realizations achieves an uncertainty smaller than that bound. This supports our contention that Eq. 1 represents a true lower bound for VFA T1 measurement uncertainty, irrespective of the number of flip angles used. For the case of N evenly distributed flip angles, shown in Figure 2, we find that the minimum error curve is simply a scaled version of the theoretical curve given by Eq. 1. The relative SNR at constant acquisition time, plotted in the right panel, demonstrates that the two flip angle case is most noise efficient, with relative SNR decreasing monotonically with increasing N.

References: **1**. MC Schabel *et al., Phys. Med. Biol.* 2008 in press. **2**. SL Deoni *et al., Magn. Reson. Med.* 2003 **49**, 515-526. **3**. HZ Wang *et al*., *Magn. Reson. Med.* 1987 **5**, 399-416.