Analytical Equation of State for Rapid and Direct Quantification of Longitudinal Relaxation Time (T1) in Look-Locker Sequences

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Introduction:
Quick and accurate measurement of the longitudinal relaxation time T_1, has become increasingly important to quantitatively estimate tissue physiological parameters such as perfusion, capillary permeability, and the volume of extravascular-extracellular space using R_1 (R_1=1/ T_1) maps in dynamic contrast-enhanced MRI (DCE-MRI) [1-5]. In the past, we have used an imaging variant of the Look-Locker sequence, the T One by Multiple Readout Pulses (TOMROP) sequence, for estimates of the temporal variation of contrast agent concentration in tissue and blood [6]. The Look-Locker (LL) sequence provides accurate T_1 estimates, with the advantages of shorter acquisition time, and a wide range of sampling times post-inversion [7].

In the Look-Locker experiment, the relationship between T_1 and the acquired signal in the Bloch equation is fairly complex. Finding the T_1 value for the LL is generally accomplished by some form of multi-dimensional curve fitting used to estimate a set of unknown parameters (T_1, tip-angle, M_0, etc) in the LL signal [7]. However, these fitting methods are sensitive to initial values (initial guesses) and a biased estimation of one parameter will bias estimates of the other parameters. In this study, an analytical equation of state for LL signals is derived and presented as an unbiased estimator of T_1. This estimator was tested by simulating the LL signal at different levels of SNR and results of its application to the DCE experimental data were compared with the T_1 maps estimated by conventional methods and the values of T_1 reported by literature.

$$S_n = K\{(M_{init} - M_{ss})\exp[-(n-1)\tau / T_1^*] + M_{ss}\}\sin\theta \exp(-T_E / T_2^*) \quad \textbf{(1)}$$

$$M_{ss} = M_0 \frac{1-\exp(-\tau/T_1)}{1-\cos(\theta)\exp(-\tau/T_1)}, \quad M_{init} = M_0 + (-EM_{relax} - M_0)\exp(-\Delta t / T_1) \quad \textbf{(2)}$$

$$M_{relax} = M_0 + (M_{ss} - M_0)\exp(-T_{relax}/T_1), \quad S_1 = KM_{init}\sin\theta\exp(-T_E/T_2^*) \quad \textbf{(3)}$$

$$S_N = KM_{SS}[(\frac{M_{init}}{M_{SS}} - 1)\xi_N + 1]\sin\theta\exp(-T_E/T_2^*), \quad 1/T_1^* = 1/T_1 - \ln(\cos\theta)/\tau \quad \textbf{(4)}$$

$$S_n = (\frac{S_1 - S_N}{1-\xi_N})\xi_n + (\frac{S_N - S_1\xi_N}{1-\xi_N}) \quad n = \{1,2,3,..N\} \quad \textbf{(5)}$$

$$\Psi(T_1) = \alpha\,\Omega^N + \beta\Omega^{N-1} + \gamma\,\Omega + \eta = 0 \quad \textbf{(6)}$$

$$\alpha = [N\bar{S} - S_N - (N-1)S_1](\cos\theta)^N$$

$$\beta = -N(\bar{S} - S_1)(\cos\theta)^{N-1}$$

$$\gamma = -N(\bar{S} - S_N)\cos\theta$$

$$\eta = [N\bar{S} - S_1 - (N-1)S_N]$$

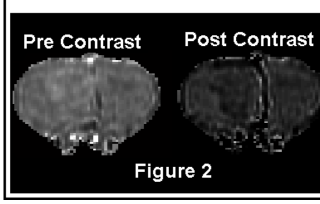
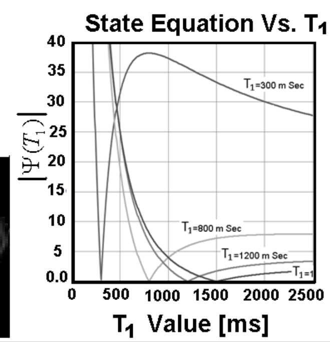


Figure 2

Figure 1



Theory: Equations 1 to 4 describe the TOMROP signal S_n and its related components acquired following the n^{th} excitation pulse (n ∈ [1 N]). Where θ denotes small tip angle excitation and τ denotes the time duration between excitations for the same slice, *K* is a proportionality coefficient dependent on detection efficiency, M_{init} is the longitudinal magnetization just prior to the first small tip angle pulse, and M_{ss} is the steady state longitudinal magnetization. Where Δt is the time delay between the inversion pulse and the first excitation pulse for a given slice, T_{relax} is the delay between the final excitation for a given slice and the inversion pulse for the next view, and *E* represents the efficiency of the inversion pulse. If ξ_n denotes $exp[-\tau(n-1)/T_1^*]$, using a recursive relationship ($\xi_{n+1} = \xi_n \exp(-\tau . R_1^*)$) between ξ_n and ξ_{n+1}, equation 5 can be derived from equations 1 to 4. By summing both sides of equation 5 from 1 to N and considering the fact that $\Sigma\xi_n$ can be written as a function of ξ_N, an equation of state can be written as Eq. 6. This equation, which is not an explicit function of T_2*, *E*, M_0, and *K*, relates T_1 to observable (measureable) parameters of the signal. Therefore, this equation is valid for all ranges of T_1 regardless of the other parameters such as T_2*, *E*, M_0, *K*. Equation of state consists of a set of measurable signal parameters (S_1, S_N and $\bar{S}$) and a set of known pulse sequence parameters (θ, N, and τ). This equation is the central equation in this study and is called a state equation since relates the T_1 parameter to the other parameters through a set of observable coefficients. Figure-1, illustrates the absolute value of $\psi(T_1)$ versus T_1 for four different T_1 values. As shown in this figure, each curve starts with a positive part and finally ends in a negative part (ascending parts), crossing the abscissa just once. This ensures that $\psi(T_1)$ is a well-behaved convex function. Therefore a simple bracket root finder can easily and rapidly find the root of $\psi(T_1)$.

Results and discussion: In this study an analytical equation of state is derived and extracted from Look-Locker (LL) inversion recovery formula to construct an accurate algorithm for direct and rapid quantification of longitudinal T_1 relaxation time. This explicit and non-empirical equation is formed by a set of measureable signal parameters and a known set of signal acquisition parameters. The derived equation is employed in a simple root finder to construct an accurate and fast algorithm for calculating the longitudinal relaxation time for the LL signal. To illustrate and test the method's accuracy for analysis of the experimental LL data, it was also applied to the LL sequences acquired from 13 animals with 9L tumor (see figure 2). Experimental results of the proposed method for all 13 animals were also compared to the results of the conventional method (Simplex method with least square fitting). Results imply that the proposed and conventional methods are highly correlated (r=0.81, p<0.0001) and also is agreement with literature value for T_1. Therefore the proposed method has a very good potential to be used as a fast and accurate T_1 map or delta R_1 map estimator from LL data in DCE studies which play an important role in quantification of physiological parameters.

References:

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