Procrustes Analysis of Diffusion Tensor Data

D. Zhou[1,2], I. L. Dryden[1,2], A. Koloydenko[3], and L. Bai[2,4]

[1]School of Mathematical Sciences, University of Nottingham, Nottingham, United Kingdom, [2]CMIAG Research Group, University of Nottingham, Nottingham, United Kingdom, [3]Mathematics Department, Royal Holloway University of London, London, United Kingdom, [4]School of Computer Science and IT, University of Nottingham, Nottingham, United Kingdom

Introduction: Diffusion tensor imaging (DTI) is becoming increasingly important in clinical studies of diseases such as multiple sclerosis and schizophrenia, and also in investigating brain connectivity. Hence, there is a growing need to process diffusion tensor (DT) images within a statistical framework based on appropriate mathematical metrics. However, the usual Euclidean operations are often unsatisfactory for diffusion tensors due to the symmetric, positive-definiteness property. A DT is a type of covariance matrix and non-Euclidean metrics have been adapted naturally for DTI processing [1]. In this paper, Procrustes analysis has been used to define a weighted mean of diffusion tensors that provides a suitable average of a sample of tensors. For comparison, six geodesic paths between a pair of diffusion tensors are plotted using the Euclidean as well as various non-Euclidean distances. We also propose a new measure of anisotropy -Procrustes anisotropy (PA). Fractional anisotropy (FA) and PA maps from an interpolated and smoothed diffusion tensor field from a healthy human brain are shown as an application of the Procrustes method.

Theory: Procrustes analysis is a statistical method to estimate the mean of a set of shapes [2]. Consider N diffusion tensors $D_1, \ldots, D_N$. To ensure the positive definite property of D_i, we use a new parameterisation, i.e. $D_i = Q_i Q_i^{\mathrm{T}}$, where Q_i is a general 3x3 matrix in the real space. The weighted Procrustes sum of squares is defined as

$$PSS(Q_1, \ldots, Q_N) = \inf_{R_i \in O(3)} \sum_{i=1}^{N} w_i \, \| Q_i R_i - \sum_{j=1}^{N} w_j Q_j R_j \|^2, \quad \sum_{i=1}^{N} w_i = 1 \qquad (1)$$

where $\|X\| = [\mathrm{trace}(X^T X)]^{1/2}$ is the Euclidean norm, weights $w_i \geq 0$ are proportional to a decreasing function of distance between voxels and R_i are orthogonal matrices. Note that Q_i and $Q_i R_i$ result in the same model, i.e. $D_i = Q_i Q_i^{\mathrm{T}} = (Q_i R_i)(Q_i R_i)^{\mathrm{T}}$. Let $Q_P = \sum w_i Q_i R_i^*$ and R_i^* are the solution of Eq.(1), $i=1,\ldots,N$. Then the weighted Procrustes mean of $D_1,\ldots,D_N$ is given by $\hat{\mu}_P(D_1,\ldots,D_N) = Q_P \, Q_P^{\,T}$.

We can also define a scaled distance function - the full Procrustes distance between two tensors: $d_P = \inf_{\beta, R_1} \| \beta Q_1 R_1 - Q_2 \|$ where $\beta > 0$ is a scalar. By computing the full Procrustes distance between a diffusion tensor and isotropy, we can derive a new measure of anisotropy called the **Procrustes anisotropy** (PA):

$$PA = \sqrt{3/2} \times d_P(Q_1, I_{3\times 3}) = \sqrt{3 \sum_{i=1}^{3} (\sqrt{\lambda_i} - \overline{\sqrt{\lambda}})^2 / (2 \sum_{i=1}^{3} \lambda_i)}, \quad \text{where } \overline{\sqrt{\lambda}} = \sum_{i=1}^{3} \sqrt{\lambda_i} / 3 .$$

Note that $0 \leq PA \leq 1$ with $PA=0$ indicating isotropy and $PA \approx 1$ indicating strong anisotropy.

Applying the Procrustes method to DT data processing, we consider smoothing and interpolation. To smooth a diffusion tensor D_k at voxel $V_k = (x_k, y_k, z_k)$ from a DT dataset Ω_D, we calculate the weighted Procrustes mean $\hat{\mu}_P(D_k, D_{k1}, \ldots, D_{kM})$ where D_{ki} are the neighbours of D_k, i.e. $D_{ki} \in \Omega_D$ and Euclidean distance between voxels V_k and V_{ki} is $d_E(V_k, V_{ki}) = \| V_k - V_{ki} \|^2 \leq a$ and $a \geq 0$. The interpolation of $D_1, \ldots, D_N$ can be obtained by calculating $\hat{\mu}_P(D_1,\ldots,D_N)$, and is allocated in a subvoxel $SV=(x,y,z)$. To consider the contribution of an individual diffusion tensor at the voxel, we let w_i be proportional to a function of d_E. E.g., for computing the smooth of D_k, we define $w_0, w_1, \ldots, w_M$ corresponding to $D_k, D_{k1}, \ldots, D_{kM}$ as w_i is proportional to $\exp\{-A d_E^2(V_k, V_{ki})\}$ where $A > 0$, i.e., exponential weights.

Results: Consider interpolations between two diffusion tensors with different metrics. Fig.1 shows six geodesic paths between two tensors (in red) differing in orientation. The Euclidean metric is problematic (see Fig.1 a) due to swelling of the volume[3]. In this study, geodesic paths using Procrustes and Root-Euclidean metrics are similar (Fig.1 c and f)., and there is not a large difference between the Riemannian [4] and Log-Euclidean [3] paths (Fig.1 d and e). Fig.1 b is the geodesic path using the Cholesky metric which smoothes out the swing between 3rd and 4th ellipsoids here.

We apply Procrustes analysis to process a set of diffusion MR images from a healthy human brain. The diffusion MR images were acquired using a spin echo EPI (echo planar imaging) sequence with diffusion weighting gradients applied with a weighting factor of b=1000 s/mm^2. A total of 52 interleaved contiguous transaxial slices were acquired throughout the subject's head in a matrix of 112x112 (interpolated to 224x224) with an acquisition voxel size of 1x1x2 mm. For each slice, the acquisition was repeated to acquire diffusion weighted images in 32 non-collinear gradient directions, as well as one acquisition with no diffusion weighting.

The DT model is fitted with a Bayesian method [5], and Fig.2 a shows the FA map. We smooth and interpolate (with 2 interpolations between each pair of original voxels) the DT data, and calculate the FA and PA maps shown in Fig.2 b and c. Obviously, FA and PA maps from the processed DT data are much smoother than the one without processing. Fig.2 a1, b1 and c1 are zoomed ROI from Fig.2 a, b and c respectively. The feature that the cingulum (cg) is distinct from the corpus callosum (cc) is clearer in the anisotropy maps from the processed data in Fig.2 b1 and c1 than that in Fig.2 a1. All computations presented in this paper are programmed with MATLAB (The Mathworks, Inc., R2008a.).

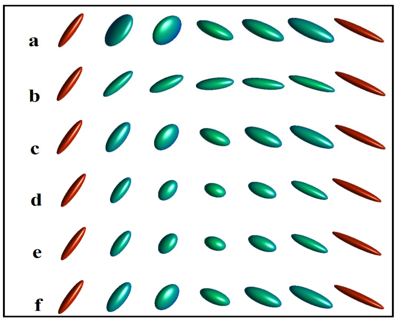

Fig.1 Geodesic paths between two tensors (in red) with Euclidean (a), Cholesky (b), Procrustes (c), Riemannian (d), Log-Euclidean (e) and root-Euclidean (f)

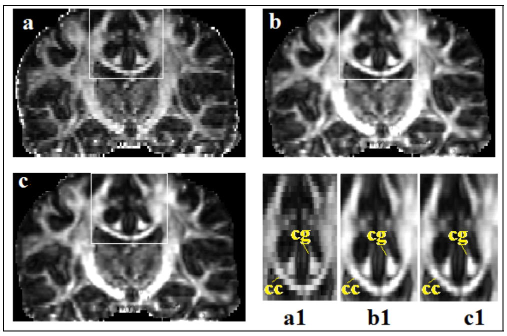

Fig. 2 a, b: FA maps from original and processed DT data, c: PA maps from processed data. a1, b1 and c1: zoomed region from a, b and c

Discussion: In this paper, we have interpolated the DT data with non-Euclidean metrics, and applied Procrustes analysis to study the anisotropy of diffusion in the human brain. Processing multiple tensors is a challenging application which would consider the contributions from different components at a voxel. Another futher application is to use the interpolated data for fibre tractograpy which provides an approriate method for investigating the structure of the human brain.

Acknowledgements: The work is supported by the European Commission FP6 Marie Curie programme through the CMIAG Research Training Network. The diffusion MR image data used in this paper is provided by the Division of Academic Radiology, University of Nottingham and Queen's Medical Centre.

References:

[1] Dryden I.L. *et al.*, 2008, submitted. [2] Dryden IL and Mardia KV, *Statistical Shape Analysis*, Wiley, 1998. [3] Arsigny V *et al.*, *Magn. Reson. Med.*, 2006, 56: 411-421. [4] Fletcher PT and Joshi S, *Sign Proces.*, 2007, 87(2):250–262. [5] Zhou D., *et al.*, *SPIE Med Imag Proc: Imag Proces*, 2008, 69142J.