Regularisation of Fractional Anisotropy using neighbourhood information

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Introduction

One of the most widely used achievements of DTI has been the parameterisation of anisotropy, which produced a new contrast mechanism between highly ordered tissue and others. The most commonly used measure of anisotropy is called fractional anisotropy (FA) [1]. Notwithstanding its successes, the traditional tensor model is a very simple model which ignores the uncertainty associated to the data caused by noise and partial volume averaging. In this study we propose the use of a Bayesian tensor regularisation algorithm which takes into consideration this uncertainty and aims at producing more reliable and robust measures of FA and other parameters derived from the diffusion tensor. The proposed method is based on an algorithm previously applied by Lu *et al.* to tractography studies [2].

Theory

Lu *et al* [2] showed that Bayes decision rule can be used to incorporate the uncertainty associated with the diffusion weighted data in the model parameters, in order to provide a better estimate of the diffusion tensor. The regularized diffusion tensor will be given by equation (1), where d represents the diffusion tensor element vector $d=(D_{xx}, D_{yy}, D_{zz}, D_{xy}, D_{xz}, D_{yz})^T$ calculated by fitting the diffusion weighted data to the diffusion tensor model [2], Σ is the covariance matrix associated to the diffusion tensor in the voxel of interest, m is the mean tensor element vector of the fibre population, Φ is the covariance matrix associated to the fibre population, and μ represents the diffusion tensor element vector estimated by the Bayes decision rule.

The covariance matrix Σ of a tensor element vector d at each voxel can be estimated directly from measured diffusion-weighted signals, experiment parameters, and signal noise profile [2,3]. The other two parameters, m and Φ, will be estimated from the neighbouring voxels to the voxel of interest. Lu [2] used the Bayes decision rule in a novel tractography algorithm to replace interpolation methods to select the best tensor for a point located anywhere within the voxel of interest, are therefore the first level neighbours corresponded to the centroids of the 8 nearest voxels to this point, and the 8 second level neighbours were selected accordingly to the fibre orientation in the point of interest. However, our aim here is to use a similar approach to refine our estimate of FA for the voxel of interest, which implies a redefinition of the sampling volume. In this study we will compare 3 different approaches:

- ***Sampling volume 1***: all 26 neighbours were used with equal weighting – $\mathbf{FA_1}$.
- ***Sampling volume 2***: a sampling volume parallel to the estimated fibre orientation in the voxel of interest is defined, by calculating the volume of each neighbouring voxel that can be accessed by any fibre coming from the voxel of interest along a specific direction (θ, ϕ), and using that volume to weight how much each neighbour should contribute for the calculation of m and Φ – $\mathbf{FA_2}$.
- ***Sampling volume 3***: the scalar product between the estimated fibre direction in the voxel of interest and the estimated direction in the neighbouring voxel is used as a measure of directional coherence, and used to weight how much each neighbour should contribute for the calculation of m and Φ – $\mathbf{FA_3}$.

$$\mu = \left(\Sigma^{-1} + \Phi^{-1}\right)^{-1}\left(\Sigma^{-1}d + \Phi^{-1}m\right) \quad (1) \qquad m = \frac{1}{\sum_{l=1}^{L} FA_l}\sum_{l=1}^{l} w[i]\times FA_l \times d_l \quad (2)$$

$$\phi_{ij} = \frac{1}{\sum_{l=1}^{L} FA_l}\sum_{l=1}^{l}\left[\left(w[i]\times FA_l \times d_{il} - m_i\right)\left(w[i]\times FA_l \times d_{jl} - m_j\right)\right] \quad i,j = 1,2,...,6 \quad (3)$$

$$CSR(FA_i) = \frac{\overline{FA_i(ROI_x)} - \overline{FA_i(ROI_y)}}{\left(\sigma_x^2 + \sigma_y^2\right)^{1/2}} \quad (4)$$

If $w[i]$ represents the weight of voxel i as defined by sampling volume 1, 2 our 3 ($w[i]$=1 for all 26 neighbours in sampling volume 1), m and Φ will be given by equations (2) and (3).

Methods

Two datasets of a healthy volunteer were acquired with a Siemens 3T Tim Trio (voxel dimensions 2.0 x 2.0 x 2.0 mm^3), using two acquisition schemes: 63 gradient directions and 1 b-value of 1000 s/mm^2, and 12 gradient directions repeated for 5 b-values equally space in the interval 0<b-value<1570 s/mm^2. FA maps for both datasets were computed by fitting the data to the diffusion tensor model [3], and then the Bayes decision rule was applied to produce three regularized FA maps, one for each sampling volume. Six regions of interest (ROIs) were drawn, corresponding to cortical grey matter (GM), thalamus (TH), parasaggital white matter (PWM), pons (P), internal capsule (IC) and splenium (S). The mean values and standard deviations (sd) calculated for FA and the three regularized maps (FA_i i=1-3) in each ROI were used to estimate the contrast-to-scatter ratio (CSR) [4] of each of these metrics of anisotropy, which is a good statistical indicator of the ability to differentiate tissues with different anisotropy levels (equation (4)).

Results and Discussion

The measured standard deviations in all ROIs for the three regularised maps are smaller than the ones observed for FA, suggesting that voxels belonging to the same population show less variability when the Bayes decision rule method is applied. For all ROIs, except the one corresponding to GM, the regularised FA using sampling volume 2 results in the most significant decrease of sd. With six ROIs is possible to calculate 15 values of CSR: GM vs TH, GM vs PWM, GM vs P, etc. All regularised FA maps show higher CSR than the simple FA map, for all of the 15 possible combinations and both acquisitions schemes (Figure 1). However, FA_2 produces the highest CSR values for 13/15 of the possible combinations of ROIs with the acquisition scheme employing 63 directions, and for 12/15 when we use the scheme employing 12 directions and 5 b-values, indicating that regularisation using sampling volume 2 produces the best overall results. In addition, the acquisition scheme using 12 directions and 5 b-values results in CSR values higher than the ones obtained with 63 directions and 1 b-value, for all combinations of ROIs and all anisotropy metrics tested, suggesting that a scheme employing multiple b-values might be more appropriate to estimate anisotropy than one employing a large number of directions but only 1 b-value.

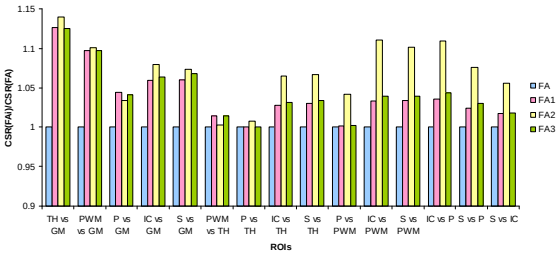

Figure 1- CSR(FAi)/CSR(FA) for each of the 15 possible combinations of ROIs (63 sampling directions).

Conclusion

We have shown that Bayes decision rule can be used to both reduce the variability observed between voxels belonging to the same population of fibres, and increase the ability for FA to differentiate between tissue types, within a range of anisotropies from FA=0.1 to FA=0.88. These two aspects may be very important when looking for differences between ROIs in different brains. Future work will include the application of this Bayes regularization technique to the study of the healthy ageing brain. In addition, this regularisation algorithm can also provide new estimates for ADC, eigenvalues, and all other parameters derived from the diffusion tensor, and the advantages of using Bayes decision rule when computing those parameters will be looked at in detail in the near future.

References

1. Pierpaoli and Basser. Magn Reson Med 1996; 36:893-906. **2.** Lu *et al*. Neuroimage 2006; 31:1061-74. **3.** Basser *et al*. J Magn Reson 1994; 103:247-254. **4.** Kingsley and Monahan. Magn Reson Med 2005; 53:911-918.