A New and Versatile Gradient Encoding Scheme for DTI: a Direct Comparison with the Jones Scheme

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Introduction. An important issue in Diffusion Tensor Imaging (DTI) is the use of a well-chosen gradient encoding scheme. Many approaches have previously dealt with simulation-based and experiment-based comparisons of encoding schemes that minimise the orientational dependence of the reliability of DTI. The most established gradient schemes to maximise SNR and minimise directional dependent bias and variations are the Jones schemes. These are based on iterative, time-consuming, numerical computations of minimal electrostatic repulsive forces between point charges on a unit sphere. Furthermore, the schemes are not exactly reproducible since they are based on a random initialisation. In order to obtain a flexible direction scheme method with the benefits of the Jones schemes, a simple geometrical scheme has been developed here. The method is very easy to implement and mimics the Jones distributions successfully.

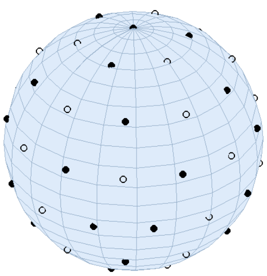

Figure 1: Discoball30 scheme. Different markers indicate the direction of the corresponding gradients.

Construction – Method. In contrast to distributing points on a circle equidistantly, distributing many points on a sphere is a highly non-trivial undertaking [4]. A strong simplification is applied here by reducing the 3D problem to the trivial 2D case. The resulting deviation from the optimal distribution is an acceptable loss, in comparison to experimental imperfections, and is later shown to be negligible. Point symmetry to the origin is naturally given by populating only one half of a sphere with points and subsequently reflecting replicas through the origin. The whole sphere is divided into regular slices by a constant zenith angle increment. Similarly, the resulting circumferences of the upper hemisphere can be divided equidistantly using constant azimuthal angle increments for each slice, respectively. Since the circumference increases by $\sin(\theta)$ (from the north pole at $\theta=0$ to the equator at $\theta=\pi/2$), the number of points distributed equidistantly on a circumference at an arbitrary $\theta>0$ must be equal to $2N\sin(\theta)$, where N is the number of predefined slices. Obviously, this equation can only be exact for the equator. For all other cases the number of points per slice must be rounded. This leads to certain total numbers of directions, i.e. 1, 3, 6, 11, 17, 23, 32, etc. Implementation of a few additional rules enables one to obtain arbitrary numbers: an expression for the number of zenith angles at a predefined total number of gradient directions is derived, $N=\sqrt{[(N_{total}-1+\pi/2)\pi/2]}$, which is then rounded and used as a starting point for constructing the scheme. The resulting total number is then compared to the predefined total number. Missing directions are added or surplus directions are subtracted in a way that accounts for the most uniform distribution. Most appropriately, the developed gradient schemes are entitled DISCOBALL schemes (*DIrection SCheme Obtained By ALigning points on Latitudes*). Figure 1 shows a Discoball30 scheme as an example.

Scheme	#	*cond*	*<σ(FA)> ± std*
Tetrahedral*	6	9.15	0.0613 ± 9.6%
Eff. dual-gradient*	6	2.00	0.0356 ± 2.7%
Eff. Icosahedral6*	6	1.58	0.03447 ± 3.1%
Jones30	30	1.59	0.03453 ± 1.0%
Discoball30	30	1.60	0.03450 ± 1.0%
Jones60	60	1.58	0.02999 ± 0.7%
Discoball60	60	1.58	0.02999 ± 0.6%

***Table 1:** Condition number and averaged standard deviation as a measure of mathematical quality and experimental accuracy for several encoding schemes (*: 5 replicas for a total of 30).*

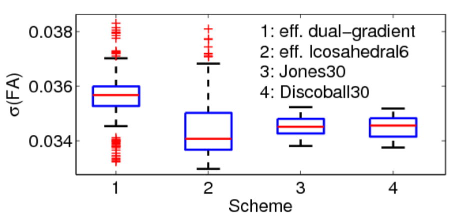

***Figure 3:** Box-whisker-plot summarising the distribution of the mean standard deviation (red crosses: outliers)*

Analysis – Method. Following the method by Skare et al. proposed in 2000 [5], adopted by Jones in 2004 [2] and Landmann et al. in 2007 [3], Monte Carlo simulations were performed using MATLAB including 10,000 repetitions of diffusion tensor estimations from noisy measurements (SNR=10.6) with 220 differently orientated, cigar shaped diffusion tensors, each possessing four different degrees of anisotropy (FA=0, 0.13, 0.71, 0.89). Analysing several encoding schemes concerning FA variation, Skare's results were reproduced for the Tetrahedral, the widely used effective Icosahedral6, and the Jones30 scheme, which was used by Skare and Landmann [5,3]. A direct comparison of Jones30 and Discoball30 scheme is presented here to demonstrate the equality of both schemes.

Results. Although not depicted here, comparison of Coulomb forces between the point charges of DISCOBALL schemes and of the Jones schemes for a range of N_{total} = 1 to 64 indicates that DISCOBALL schemes perform almost as well as the optimal Jones schemes. Table 1 puts the DISCOBALL schemes into context with the commonly used schemes. Concerning their resulting condition numbers, the Jones and DISCOBALL schemes perform similarly. Figure 2 compares the mean standard deviations over all Monte Carlo repetitions vs. tensor orientation. Additional averaging over all tensor orientations yields a measure for the overall accuracy of the scheme. The corresponding standard deviation indicates the accuracy fluctuation over all tensor orientations. Both are also listed in table 1. The box-whisker-plot in figure 3 provides a similar comparison of the mean standard deviation.

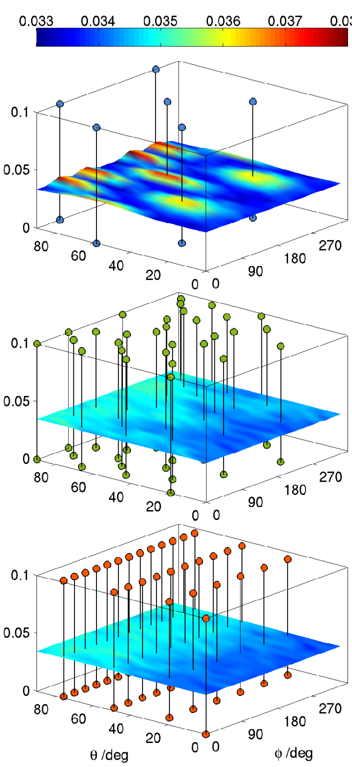

***Figure 2:** Standard deviation of FA over all repetitions averaged over all FA's. Vertical lines indicate the gradient directions. Top: eff. Icosahedral6 (x5), Centre: Jones30, Bottom: Discoball30.*

Conclusion. The proposed DISCOBALL gradient encoding schemes perform as well as the computationally demanding Jones schemes. The approach for arbitrary numbers is fast, unique, and well-suited for direct implementation into the MRI sequences instead of using look-up tables.

References: [1] Jones, D. K. et al., A., *Magnetic Resonance in Medicine,* **1999***, 42*, 515-525; [2] Jones, D. K., *Magnetic Resonance in Medicine,* **2004***, 51*, 807-815; [3] Landmann, B. A. et al. *Neuro Image,* **2007***, 36*, 1123-1138; [4] Saff, et al., *The Mathematical Intelligencer,* **1997***, 19(1)*, 5-11; [5] Skare, S. et al, *Journal of Magnetic Resonance,* **2000***, 147*, 340-352