A New Quality Measure for Gradient Encoding Schemes

S. C. Mang[1,2], D. Gembris[2,3], W. Grodd[1], and U. Klose[1]

[1]Section Exp. MRI of CNS, Diagnostic and Interventional Neuroradiology, Tübingen, Germany, [2]Institute for Computational Medicine, University of Mannheim, Mannheim, Germany, [3]Bruker Biospin MRI GmbH, Ettlingen, Germany

Introduction: The limitation of the 2nd order diffusion tensor model in the representation of voxels containing more than one dominant fiber direction have led to the development of more advanced representations. Here the focus is especially on higher order diffusion tensor models [1,2]. These more advanced diffusion models require datasets which provide diffusion information for a larger number of gradient encoding directions. The dependence of the 2nd order tensor quality on the gradient direction set used in the data acquisition has been previously investigated (for example [3-5]), but higher order tensor models pose different demands on the diffusion data set. Here we investigate the effect of differently distributed sets of gradient encoding directions, so called gradient encoding schemes (GES) on higher order tensor models. For this purpose we propose a new measure for the quality of GESs, the investigation of the "signal deviation".

Methods: GES Quality Measure: The signal deviation η is estimated by comparing the signal $S_g(I)$ with the signal $S_g(T)$ for each direction in the GES and averaging the differences (III in Fig.1). To account for rotational dependence of the GES quality, we did rotate the input tensor D_I, did evaluate η for each rotation and used the mean η over all rotations as quality measure. A GES was considered to be of "good quality" if it had a low mean η and low corresponding standard deviation. The signal deviation is not only able to show the influence of the GES, when η is evaluated for a single diffusion model and different GES, but can also be used to evaluate the accuracy of the representation for different diffusion models if the same GES is used to compute η for different models. Evaluated GES: We did evaluate this new GES quality measure for GES that are based on mesh refinements of a regular icosahedral grid (Icosa [5]), pair wise and individual force-minimization (ForcePairs [6] and ForceSingle [7]), condition number minimization (Cond [4]) and analytical formulas (Ana1 [8] and Ana2 [9]). Evaluated Tensor Models: The signal deviation was evaluated for the 2nd order tensor model [10], the 4th order tensor proposed by Ozarslan and Mareci [2] and the higher order tensor hierarchy proposed by Liu et al [1].

$$I)\, S_g(I) = S_0 \exp(-BD_I) + noise; \qquad II)\, S_g(T) = S_0 \exp(-BD_T)$$

$$III)\, \eta = \frac{1}{N_e} \sum_{k=1}^{N_e} \frac{\left| S_{g_k}(T) - S_{g_k}(I) \right|}{S_0}$$

Figure 1: In I the input signal $S_g(I)$ for direction g is derived from a chosen input tensor D_I, with S_0 the un-weighted signal and B the estimation matrix, which contains the information on the GES and the diffusion weighting (b-factor). In II the signal for testing the representation quality $S_g(T)$ is computed from the tensor D_T that was fitted to $S_g(I)$. The signal deviation η is calculated with III, where N_e is the number of encoding directions.

Results: The signal deviation was evaluated for different noise levels and numbers of directions. In Fig.2 exemplary results for an evaluation of GES with 21 directions, which is the smallest possible N_e for the higher order tensor hierarchy up to tensor order four, are shown. The chosen input tensor D_I was a higher order tensor hierarchy representation of a voxel containing two orthogonally crossing fibers. The results of an evaluation without noise are presented in Fig.2, because they show the individual differences between the GES most clearly. The mean η and corresponding standard deviation are scaled (scaling factors are given in Fig.2) to allow an inter-model comparison even though the absolute values for the individual diffusion tensor models are of different order of magnitude. For ForcePairs [6] for example the mean η (standard deviation) for the 4th order tensor is 3.342 (0.007), for the 2nd order tensor 0.117 (2.61e-4) and for the higher order tensor hierarchy 1.53e-3 (5.96e-5). The evaluation of the mean η (bars in Fig.2) shows that the Cond GES has a considerably higher mean compared to the other GES independent of the tensor model. The difference between the mean values for the other GESs is not as clear. The standard deviation (error bars in Fig.2) also shows a strong dominance of the Cond scheme but allows differentiation between the results for the other GESs. The analytical GES (Ana1 and Ana2) and ForceSingle show a larger standard deviation than ForcePairs and Icosa, which can be observed best in the scaled tensor hierarchy results in Fig.2. A slight advantage of ForcePairs over Icosa is observable for both higher order tensor models but not for the 2nd order tensor.

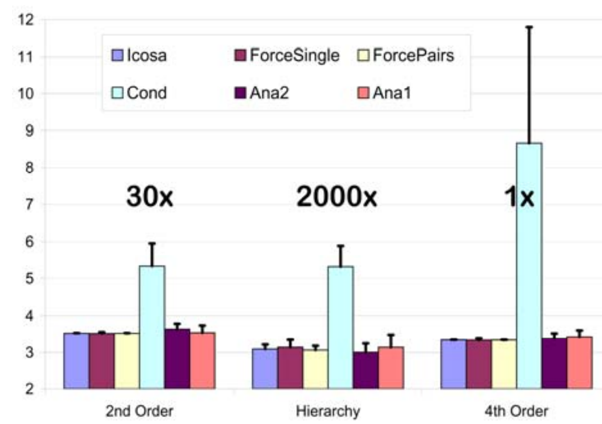

Figure 2: The scaled mean signal deviation is plotted for all evaluated GES and tensor models. The tensor model dependent scaling factors are given in the plot.

Discussion: The here introduced signal deviation is a GES quality measure that is directly applicable to the 2nd order diffusion tensor and the here evaluated higher order tensor models. The signal deviation η can also be evaluated for other diffusion models so that good quality GESs for any model can be found. The results from the evaluation of the 2nd order diffusion tensor correspond to the results presented in earlier studies [3-5]. The ForcePairs and Icosa GES clearly outperform all other GESs in our evaluation. For higher order tensor models ForcePairs shows a slightly lower standard deviation of η. ForcePairs is therefore the most advantageous GES for the here evaluated diffusion models.

References:

[1] Liu C et al. Characterizing non-Gaussian diffusion by using generalized diffusion tensors. MRM 2004;51(5):924–937. [2] Ozarslan E, Mareci TH. Generalized Diffusion Tensor Imaging and Analytical Relationships Between Diffusion Tensor Imaging and High Angular Resolution Diffusion Imaging. MRM 2003;50:955–965. [3] Jones DK. The effect of gradient sampling schemes on measures derived from diffusion tensor MRI: a Monte Carlo study. MRM 2004;51(4):807–815. [4] Skare S et al. Condition Number as a Measure of Noise Performance of Diffusion Tensor Data Acquisition Schemes with MRI. JMR 2000; 147(2):340–352. [5] Batchelor PG et al. Anisotropic Noise Propagation in Diffusion Tensor MRI Sampling Schemes. MRM 2003;49(6):1143–1151. [6] Jones DK et al. Optimal Strategies for Measureing Diffusion in Anisotropic Systems by Magnetic Resonance Imaging. MRM 1999; 42(3):515–525. [7] Fliege J, Maier U. The distribution of points on the sphere and corresponding cubature formulae. IMA J NUMER ANAL 1999;19(2):317-334. [8] Wong ST, Roos MS. A strategy for sampling on a sphere applied to 3d selective rf pulse design. MRM 1994;32(6):778–784. [9] Saff EB, Kuijlaars ABJ. Distributing many points on a sphere. MATH INTELL 1997;19(1):5–11. [10] Basser PJ, Pierpaoli C. A Simplified Method to Measure the Diffusion Tensor from Seven MR Images. MRM 1998;39:928–934.