An inverse design method for 3D toroidal gradient coils

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Introduction: Traditional gradient coil design is typically constrained to cylindrical, planar, spherical or conical surfaces. Gradient coil geometry is important in arriving at optimal trade-offs between gradient homogeneity, coil efficiency and inductance [1]. Additional concerns such as the induction of eddy currents and peripheral nerve stimulation, patient claustrophobia and acoustic noise have dictated novel coil structures and design methods (*eg* [2], [3]). However, these methods ordinarily demand that the geometry of the gradient coils be specified prior to optimisation. While *et al.* [4] present a method for designing fully 3D transverse gradient coils, in which the precise geometry is obtained as part of the optimisation process. This method solves for a 3D current density vector and obtains coil windings using a priority streamline seeding technique. Results are found to display an interesting general geometric form involving sets of closed loops plus spiral-type coils, and lie approximately on the surfaces of sets of elliptical tori. However, despite displaying excellent gradient homogeneity, coil efficiency is low due to the small number of windings and increasing this number results in an unattractive design in terms of manufacturability. The aim of the subsequent work presented here is to use the 3D windings of [4] as a guide for choosing appropriate elliptical torus current density surfaces for which to repeat the gradient field optimisation. It is envisaged that this will provide a transverse gradient coil with a similar optimal geometric form to that of [4], but one that is easy to manufacture via machine etching and should also offer much greater freedom in regards to optimising the trade-off between coil efficiency, inductance and field error.

Method: The model we consider involves K elliptical tori of radii a_k ($k = 1:K$), with semi-major axes b_{k1} and semi-minor axes b_{k2}, lying coaxially with the z-axis and centred on the points $(x,y,z) = (0,0,c_k)$. On each torus there exists a surface current density vector $\boldsymbol{j}_k(\theta',\phi')$ (A/m), which is represented by Fourier series in the torus coordinates θ' and ϕ', and must be divergence free. A solution is sought for the Fourier coefficients such that the current density on the toroidal surfaces induces a linear magnetic field on the surface of some interior spherical target region of radius α, centred at the origin. To account for the ill-conditioning of the corresponding integral equation, a regularisation strategy is implemented in which the field error is minimised in conjunction with the power of the coil. A shielding constraint may also be applied on some external target surface. The current density solution is discretised by contouring the associated streamfunction, and coil windings are analysed in terms of field error, efficiency, inductance, Lorentz force and torque.

Fig. 1: 24 windings used to approximate the 3D current density $\boldsymbol{J}(r',\theta',z')$, for an unshielded x-gradient coil in While *et al.* [4].

Results: In While *et al.* [4] the greatest success was achieved for an unshielded whole-body system comprising six sets of closed loops and spiral-type coil windings, and this result is reshown in Fig. 1. Therefore, for the present work, the number of tori surfaces was chosen to be $K = 6$ and the remaining geometric parameters $(a_k, b_{k1}, b_{k2}, c_k)$ governing the shape of the tori were obtained using a least squares type fitting to the vertices of the 3D coil windings in Fig. 1. As an example, Fig. 2(a) displays a discretised set of 128 windings obtained for the six tori of an unshielded system, with DSV radius $\alpha = 0.15$ m and gradient strength 50 mT/m. The distribution of current depends largely on the regularising parameter λ_P. Fig. 2(b) shows the winding pattern for the first torus plotted on the (θ',ϕ') plane and displays a familiar fingerprint type pattern. In comparison, we see that in the portion of the coils in Fig. 1 where the spirals are tightly wound there is strong axial current, which is also observed in the same portion of the coils in Fig. 2(a). Similarly, in the portion of the coils in Fig. 1 where the spirals open up (near the plane y = 0) there is strong azimuthal current, which is also observed in the same potion of the coils in Fig. 2(a). The coils displayed in Fig. 2(a) correspond to a 5% field error DSV of radius 0.202 m, have a coil efficiency of 67.4 µT/A/m and an inductance of 634 µH. However, efficiency and inductance can be improved considerably at the expense of gradient homogeneity by increasing λ_P and hence the impact of the minimum power constraint. Self-shielding can be included in the design yielding a 95% reduction in stray fields at the expense of a 50% drop in efficiency. The Lorentz forces over each half of each torus are found to be negligible (order 10^{-10} N) suggesting considerable force cancelling is achieved. The associated torque is found to have a dominant z-component of 0.2-0.3 Nm/A/T, which is appropriate for systems without direct torque balancing. The method was also applied to the design of a transverse head gradient coil by simply manipulating the geometrical parameters governing the individual tori. One such design, similar to Fig. 3, but with 136 windings, had an efficiency of 183.4 µT/A/m, an inductance of 115 µH, a 5% field error DSV of radius 0.102 m and similarly small Lorentz forces and torque. As for the whole-body coil, these parameters could be traded considerably by changing the number of coil windings or altering λ_P. For example, Fig. 3 displays a coil with only 52 windings that had an efficiency of 114.4 µT/A/m and an inductance of only 28 µH, for a similar field error.
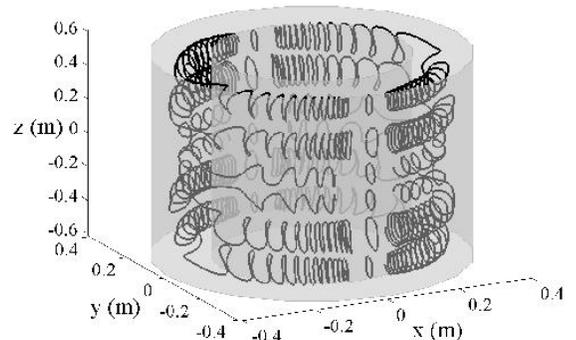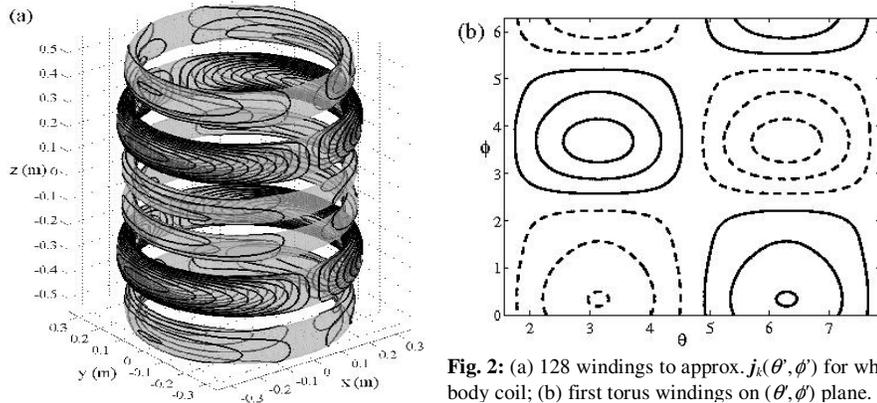

Fig. 2: (a) 128 windings to approx. $\boldsymbol{j}_k(\theta',\phi')$ for whole-body coil; (b) first torus windings on (θ',ϕ') plane.
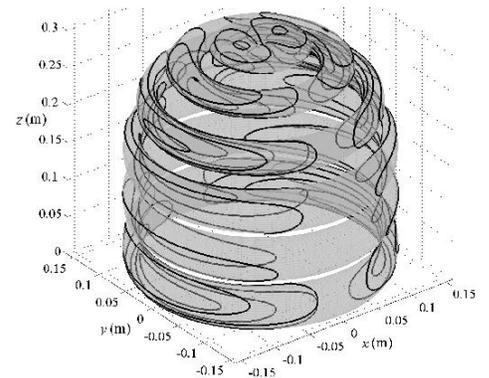

Fig. 3: 52 windings to approx. $\boldsymbol{j}_k(\theta',\phi')$ for head coil.

Conclusion: An inverse method has been presented for the theoretical design of transverse gradient coils based on the optimal geometry of a fully 3D solution. Coil windings were chosen to reside on a set of toroidal surfaces and results displayed high coil efficiency, low inductance, high gradient homogeneity and good force balancing. Additional perceived benefits include improved thermal performance via greater access to cooling mechanisms and a reduction in acoustic noise.

References:
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