High order stream function method to automatically design MRI gradient coil

F. Jia[1], Z. Liu[1], A. M. Welz[2], M. Zaitsev[2], J. G. Korvink[1], and J. Hennig[2]

[1]Dept. of Microsystems Engineering – IMTEK, University of Freiburg, Freiburg, Germany, [2]Dept. of Diagnostic Radiology, Medical Physics, University Hospital Freiburg, Freiburg, Germany

Introduction

Over the past 20 years many papers have discussed theoretical design methodologies for MRI gradient coils. Lemdiasov et al. [1] presented a method to design a gradient coil on an arbitrary surface. The current-carrying surface is discretized using a triangular mesh, and the current density value on the surface can be obtained by optimizing a cost function related to the desired magnetic field. This is an attractive method because the current-carrying surface could be arbitrary instead of a regular cylindrical surface. However, there is a discretization error when one uses low-order Lagrange type elements to approximate the current-carrying surface which has spatially-varied surface curvature. At the same time, it is difficult to specify a continuous current vector along the inter-element boundary, because the normal vector on the discretized Lagrange element is discontinuous. Even though these errors could be reduced by refining the surface mesh, this remedy may meet the bottleneck of the computational cost for a large-scale design procedure.

In this paper, we present a **high-order discretization method** to design the cylindrical gradient coil. The cylindrical surface can be **exactly discretized** using a high-order triangular mesh when the surface is expressed using a cylindrical coordinate system. The magnetic field is calculated using the Biot-Savart law where the surface current density is expressed using a stream function and the surface integration is implemented using **surface numerical integration** based on the shape functions of the finite element.

Theory

The optimization objective used in this paper is the least square type function

$$F = \sum_{i=1}^{k} w_i (B_z(x_i, y_i, z_i) - B_{des,z}(x_i, y_i, z_i))^2 \quad (1)$$

where $B_{des,z}(x_i, y_i, z_i)$ is the z-component of the desired magnetic field, (x_i, y_i, z_i) are the coordinates of the sampling points in the region of interest (ROI), and w_i is the weight coefficient at the point (x_i, y_i, z_i). The magnetic field B_z is calculated using the Biot-Savart Law in a cylindrical coordinate system:

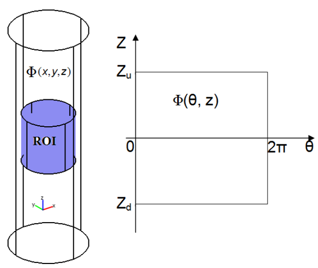

Figure 1. current-carrying surface and its expression in a cylindrical coordinate system

$$B_z(x_i, y_i, z_i) = \int_{z_d}^{z_u}\int_0^{2\pi} \frac{-J_y(\theta,z)(r_0\cos(\theta)-x_i) - J_x(\theta,z)(r_0\sin(\theta)-y_i)}{((r_0\cos(\theta)-x_i)^2 + (r_0\sin(\theta)-y_i)^2 + (z-z_i)^2)^{3/2}} r_0 \, d\theta dz \quad (2)$$

where J_x and J_y are the x and y components of the surface current density respectively and r_0 is the radius of the current-carrying surface $[0,2\pi]\times[z_d,z_u]$, (figure 1). The surface current density can be expressed using the stream function as:

$$J = \nabla \times (\Phi \vec{n}) \quad (3)$$

where $\vec{n}$ is the normal vector on the cylindrical surface. In the cylindrical coordinate system, equation (3) can be expressed as

$$\nabla \times (\Phi \vec{n}) = (0, \partial\Phi(\theta,z)/\partial z, -\partial\Phi(\theta,z)/(r_0 \partial\theta))^T \quad (4)$$

therefore, the x and y components of the surface current density are

$$J_x(\theta,z) = -\frac{\partial\Phi(\theta,z)}{\partial z}\sin(\theta), J_y(\theta,z) = \frac{\partial\Phi(\theta,z)}{\partial z}\cos(\theta) \quad (5)$$

The current-carrying surface is discretized using a triangular mesh and the value of the stream function is interpolated using the shape function Ψ_j of the **Argyris element** [2], i.e. $\Phi = \Sigma_j \alpha_j \Psi_j$ where α_j are coefficients. One unique advantage of **Argyris elements** is that their basis functions have continuous derivatives between adjacent mesh triangles. Therefore the continuity equation of the current is satisfied on the current-carrying surface. Using the high-order smooth interpolation strategy, the magnetic field Bz can be calculated **accurately and efficiently**.

The gradient coil optimization is an inverse problem. Generally, one needs to use the regularization technique to **avoid** the **oscillation** of the coil layout. Typically the inductance of the coil, or the magnetic energy term is combined to obtain a reasonable layout of the coil. In our example, we use the conjugate gradient method with filter technique to implement the regularization effect. In the conjugate gradient method, the sensitivity of our objective can be obtained by the following formula

$$\frac{\partial F}{\partial \alpha_j} = 2\sum_{i=1}^{k} w_i (B_z(x_i,y_i,z_i) - B_{des,z}(x_i,y_i,z_i)) \frac{\partial B_z(x_i,y_i,z_i)}{\partial \alpha_j}, \frac{\partial B_z(x_i,y_i,z_i)}{\partial \alpha_j} = \int_{z_d}^{z_u}\int_0^{2\pi} \frac{-(\partial\Psi_j/\partial z)\sin(\theta)(r_0\cos(\theta)-x_i) - (\partial\Psi_j/\partial z)\cos(\theta)(r_0\sin(\theta)-y_i)}{((r_0\cos(\theta)-x_i)^2 + (r_0\sin(\theta)-y_i)^2 + (z-z_i)^2)^{3/2}} r_0 \, d\theta dz \quad (6)$$

Numerical results

Figure (2) shows an example of a Gx gradient coil design on the cylinder surface with radius=0.045m and height=0.27m. The ROI is a cylinder with radius=0.03m and height=0.06m (figure 1). The weight w_i is 1 at all sampling points. This surface is discretized into a triangular mesh in the cylindrical coordinate system. The stream function and the coil layout (contour line of the stream function) are shown in figure 2a. The gradient strength at the center of ROI is 2.276mT/m/A.

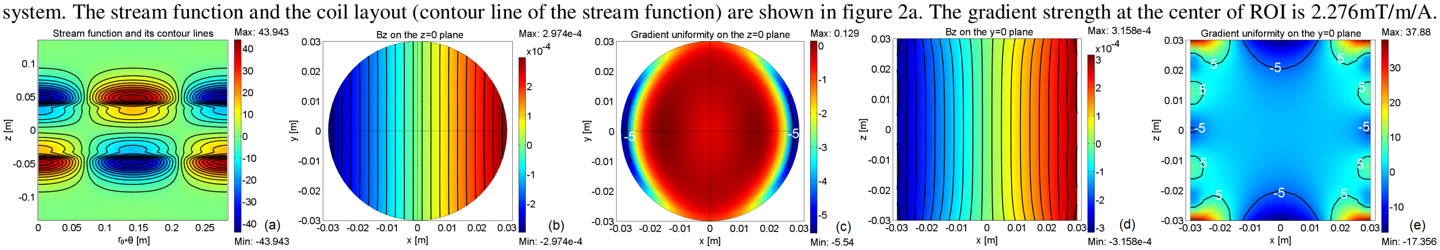

Figure 2, (a) the stream function and its contour lines. (b) and (d) magnetic flux density B_z on the z=0 and y=0 plane respectively. (c) and (e) Percentage deviation of $\partial B_z/\partial x$ from specified gradient strength; the curves depict the absolute value of 5.

Discussion and outlook

This abstract presents a **high-order stream function method** to design a **surface gradient coil** for MRI. Based on a finite element type surface discretization and numerical integration technique, one numerical example demonstrates that this method can be used to design a surface gradient coil in which the surface is convex and has no singularity. In the future, a more general current-carrying surface will be implemented and the multiple objective functions will be considered in order to extend the method to more general topographies.

Reference

1 R.A. Lemdiasov, R. Ludwig, A stream function method for gradient coil design, Concepts in Magnetic Resonance Part B Vol. 26B(1) 67-80 (2005)

2 S.C. Brenner, L. R. Scott, The mathematical theory of finite element methods, Second version, Springer-Verlag, 2002

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