Theoretical design of 3D gradient coils

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Introduction: Gradient coil design is constrained by the important coil parameters of inductance, efficiency and field homogeneity [1]. Additional secondary concerns include reducing the induction of eddy currents and peripheral nerve stimulation, patient claustrophobia and acoustic noise [2]. The search for optimal coil geometries to address various combinations of these design criteria has resulted in a wide variety of coil structures to be presented in the literature (*eg* [3], [4]). These recent designs deviate away from the established cylindrical or bi-planar, primary plus shield coil set; however, despite the associated methods often being applicable to arbitrary geometry, this geometry must still be chosen prior to the method being implemented. A method is presented here for the theoretical design of transverse gradient coils that allows full freedom to be explored within 3D solution space. That is, the coil windings are not constrained to lie on some predetermined surface, but are instead found as part of the optimisation process. Interest lies primarily with the optimum geometry of the coil windings in obtaining a linear gradient field, with coil inductance, efficiency and shielding being secondary concerns in this preliminary work.

Method: The 3D gradient coil is defined to exist within the volume between two cylinders of length $2L$, with inner radius a and outer radius b, lying coaxially with the z-axis and centred at $z = 0$. This volume carries an unknown current density vector $\boldsymbol{J}(r',\theta',z')$ (A/m^2), which is represented by Fourier series in r', θ' and z', and must be divergence free. A solution is sought for the current density Fourier coefficients such that a linear magnetic field is induced on the surface of an interior spherical target region of radius c, centred at the origin. A shielding constraint may also be applied. A regularisation strategy is implemented to overcome the ill-posed nature of the problem, whereby the power of the coil is minimised in conjunction with the field error. This method has been used previously for the design of cylindrical and biplanar surface gradient coils (*eg* [5]); however, the extra radial coordinate and radial current density component in the present 3D method greatly complicate the mathematics. In addition, for a surface current density the continuity equation allows components to be related via a streamfunction, the contours of which provide coil winding positions. However, such a convenience does not exist for a 3D current density vector and alternative methods are required. A priority streamline seeding technique developed for mapping velocity fields in fluid flow [6] was modified and used to find appropriate coil windings that would approximate the 3D current density. In this method a density map related to the current density magnitude is created and streamlines are seeded at the locations of maxima. A 3D Gaussian filter is then applied to the density map along the trace of each streamline and the process is repeated until some threshold is reached or a sufficient number of streamlines have been plotted. Prior knowledge of the symmetric nature of the current density aids this process. Ideally these coil windings would carry identical current; however, a secondary optimisation of the coil currents is applied to account for the approximate nature of the priority streamline technique and to obtain the optimum induced field.

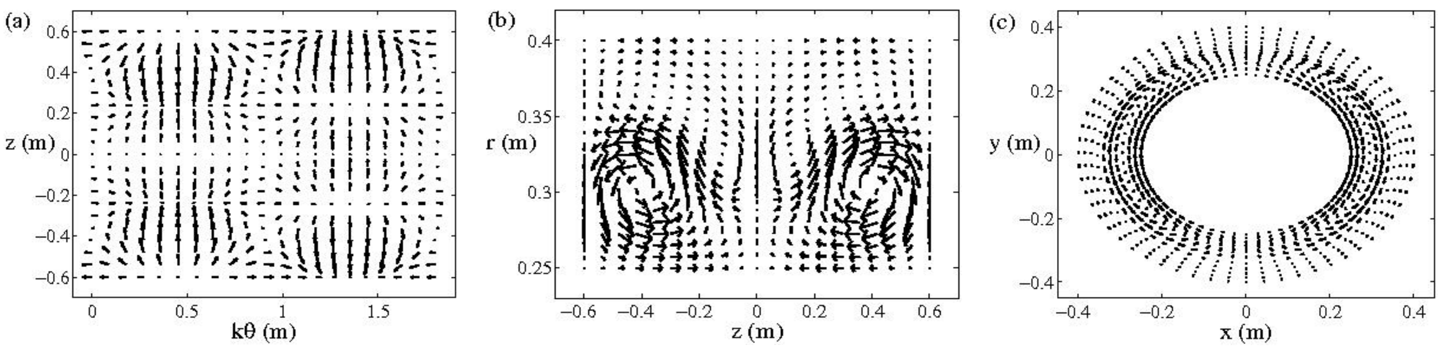

Fig. 1: Quiver plots of $\boldsymbol{J}(r',\theta',z')$ with $\lambda_P = 10^{-6}$, on: (a) the $(k\theta,z)$ plane at constant $r = k = a + (b-a)/4$; (b) the (r,z) plane at $\theta = \pi/2$; (c) the (x,y) plane at $z = L/4$.

Results: The weighting λ_P of the minimum power constraint governs the trade-off between current density complexity and field homogeneity. Fig. 1 displays quiver plots of the 3D current density vector in three orthogonal planes for a coil volume of length $2L = 1.2$ m, inner radius $a = 0.25$ m and outer radius $b = 0.4$ m, with DSV radius $c = 0.15$ m, gradient strength 50 mT/m and $\lambda_P = 10^{-6}$. In Fig. 1(a) the quiver plot is on the $(k\theta,z)$ plane at constant radius $r = k = a + (b-a)/4$. The apparent sources and sinks merely represent the flow of current density from other regions of the coil volume. Note that the form displayed in Fig. 1(a) is vastly different to the fingerprint designs of traditional cylindrical gradient coils, which share this plane. This is interesting as it suggests that these traditional coil sets are optimum only for their specific predetermined geometry, rather than in a general sense as for the present 3D solution. In Fig. 1(b) the quiver plot is on the (r,z) plane at constant $\theta = \pi/2$ and displays two main cyclic features which dominate the flow of current in this plane. In Fig. 1(c) the quiver plot is on the (x,y) plane at constant $z = L/4$ and shows strong current flowing along the inner radius of the coil volume and then returning at a higher radius in the opposite azimuthal direction. The planes in Fig. 1(b) and 1(c) are interesting as they are not available in the traditional cylindrical gradient designs. Fig. 2 displays the corresponding first eight coil windings obtained using the priority streamline seeding method. These are a combination of closed loops and spiral-type coil windings that traverse much of the azimuthal axis, near the coil ends, and clearly approximate the form of the 3D current density displayed in Fig. 1. Relaxing the minimum power constraint by choosing $\lambda_P = 10^{-10}$ increases the number of spiral features in Fig. 2 from two to six and results in improved field homogeneity (0.54% field error within DSV). Including active shielding in the model yields similar results, which suggests an inherent self-shielding property and provides additional evidence that the optimum geometry of the 3D coil windings has been found in general.

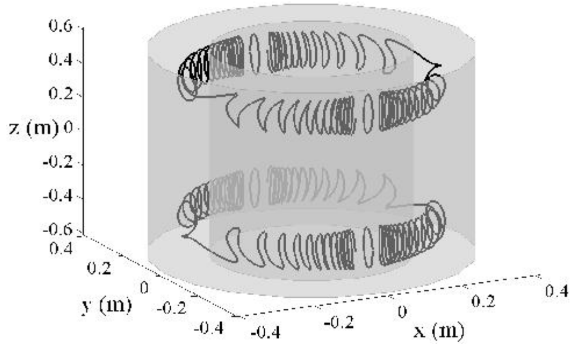

Fig. 2: 8 windings used to approximate $\boldsymbol{J}(r',\theta',z')$, with $\lambda_P = 10^{-6}$.

Conclusion: A method has been presented for the theoretical design of 3D gradient coils of optimal geometry. Whilst these coil windings appear complex in nature, it is envisaged that their form may guide future gradient structures. For instance, the closed loop and spiral-type coil windings presented here approximately lie on elliptical tori coaxial with the z-axis. Indeed, subsequent to this work, such toroidal gradient coils have been designed displaying high gradient homogeneity, low inductance, high efficiency and good force balancing, as well as offering perceived benefits to gradient cooling and patient claustrophobia.

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