Direct calculation of tissue electrical parameters from B1 maps

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Introduction: There is interest in *in-vivo* measurement of tissue electrical parameters (conductivity and permittivity) for diagnostic purposes, as malignant cells are known to have different electrical properties from normal cells [1]. Tissue conductivity is also required to estimate local heating effects in multi-channel transmit schemes. MRI based method to estimate complex permittivity was presented in [2], but the method requires iterative calculations to estimate unknown Ez and permittivity. A local formula was presented in [3], but the method requires the measurement of full magnetic field.

In this work, we derive a set of equations to directly calculate tissue electrical parameters from B_1^+ maps. The method requires only the knowledge of complex B_1^+ in a volume of constant electrical properties.

Theory: The phasor notation for time harmonic fields is used in the following derivation. From the Ampere's Law (1), we obtain the x, y and z components of the electric field as (2), (3) and (4). From the Faraday's Law (5), we obtain the x component of the magnetic field as (6). Substituting the electric field expressions (3) and (4) into equation (6), with the assumption of spatially invariant ε and σ in the local region, we obtain equation (7). From the divergence equation (8), we obtain (9). Taking the derivative with respect to x and re-arranging terms, we obtain (10). Substituting (10) into (7) and simplifying, we obtain (11) for Hx. Following similar steps, we obtain (12) for Hy. Using (13) (definition of B_1^+), and equations (11) and (12), we can write (14). Then the unknown tissue conductivity and relative permittivity can be written as a function of $B1^+$ as shown in the left box, where Re{} and Im{} are the real and imaginary components of the argument. An identical set of equations can be derived for conductivity and relative permittivity as a function of B_1^{-*}. The results indicate that tissue properties can be estimated using either transmit or receive sensitivity maps.

$$\hat{\sigma} = \mathrm{Re}\left\{\frac{1}{j\omega\mu B1+}\left(\frac{\partial^2 B1+}{\partial x^2}+\frac{\partial^2 B1+}{\partial y^2}+\frac{\partial^2 B1+}{\partial z^2}\right)\right\}$$

$$\hat{\varepsilon}_r = \frac{1}{\omega\varepsilon_0}\mathrm{Im}\left\{\frac{1}{j\omega\mu B1+}\left(\frac{\partial^2 B1+}{\partial x^2}+\frac{\partial^2 B1+}{\partial y^2}+\frac{\partial^2 B1+}{\partial z^2}\right)\right\}$$

$$\nabla\times\mathbf{H} = (\sigma + j\omega\varepsilon)\mathbf{E} \quad \text{------(1)}$$

$$Ex = \frac{1}{\sigma + j\omega\varepsilon}\left(\frac{\partial Hz}{\partial y}-\frac{\partial Hy}{\partial z}\right) \quad \text{------(2)}$$

$$Ey = \frac{1}{\sigma + j\omega\varepsilon}\left(\frac{\partial Hx}{\partial z}-\frac{\partial Hz}{\partial x}\right) \quad \text{------(3)}$$

$$Ez = \frac{1}{\sigma + j\omega\varepsilon}\left(\frac{\partial Hy}{\partial x}-\frac{\partial Hx}{\partial y}\right) \quad \text{------(4)}$$

$$\nabla\times\mathbf{E} = -j\omega\mu\mathbf{H} \quad \text{------(5)}$$

$$Hx = \frac{-1}{j\omega\mu}\left(\frac{\partial Ez}{\partial y}-\frac{\partial Ey}{\partial z}\right) \quad \text{------(6)}$$

$$Hx = \frac{-1}{j\omega\mu(\sigma + j\omega\varepsilon)}\left(\frac{\partial^2 Hy}{\partial x\partial y}-\frac{\partial^2 Hx}{\partial y^2}-\frac{\partial^2 Hx}{\partial z^2}+\frac{\partial^2 Hz}{\partial x\partial z}\right) \quad \text{------(7)}$$

$$\nabla\cdot\mathbf{B} = 0 \quad \text{------(8)}$$

$$\frac{\partial Hx}{\partial x}+\frac{\partial Hy}{\partial y}+\frac{\partial Hz}{\partial z} = 0 \quad \text{------(9)}$$

$$\frac{\partial^2 Hz}{\partial x\partial z} = -\left(\frac{\partial^2 Hx}{\partial x^2}+\frac{\partial^2 Hy}{\partial x\partial y}\right) \quad \text{------(10)}$$

$$Hx = \frac{1}{j\omega\mu(\sigma + j\omega\varepsilon)}\left(\frac{\partial^2 Hx}{\partial x^2}+\frac{\partial^2 Hx}{\partial y^2}+\frac{\partial^2 Hx}{\partial z^2}\right) \quad \text{--(11)}$$

$$Hy = \frac{1}{j\omega\mu(\sigma + j\omega\varepsilon)}\left(\frac{\partial^2 Hy}{\partial x^2}+\frac{\partial^2 Hy}{\partial y^2}+\frac{\partial^2 Hy}{\partial z^2}\right) \quad \text{--(12)}$$

$$B1+ = \frac{1}{2}\mu(Hx + jHy) \quad \text{------(13)}$$

$$B1+ = \frac{1}{j\omega\mu(\sigma + j\omega\varepsilon)}\left\{\frac{\partial^2 B1+}{\partial x^2}+\frac{\partial^2 B1+}{\partial y^2}+\frac{\partial^2 B1+}{\partial z^2}\right\} \quad \text{----(14)}$$

Methods: To validate the method, we consider B_1^+ obtained from electromagnetic simulations. The geometric model for the simulation has a 32-rung birdcage coil and a human body model. The human body model has 23 different tissue types, each with distinct electrical properties (conductivity and permittivity). The model is anatomically correct, and therefore the variation of electrical properties is consistent with the human body.

The B_1^+ distribution within the human body model is obtained by energizing the birdcage coil at 128MHz frequency, corresponding to 3.0T imaging. The steady state flux data (Bx, By) are obtained using the finite difference time domain method (xFDTD, Remcom, PA, USA), and data corresponding to three axial slices are imported into Matlab (Mathworks, MA, USA).

The B_1^+ for central axial slice and two axial slices each from superior and inferior sides are calculated in Matlab. Then the equations for estimating conductivity and permittivity are solved, using finite differences for partial derivatives. The calculations result in estimates of conductivity and permittivity for the central axial slice.

Results: The conductivity estimated from B_1^+ is shown in Fig. 1. In regions of constant electrical properties with sufficient B_1^+ samples, the equations estimate the conductivity accurately. The mesh size for the human body model is 5mm x 5mm x 5mm. Therefore, in regions where the variation of tissue properties is high, sufficient number of B1+ samples are not available in a volume of constant electrical properties. This may be overcome by performing the finite differencing operations using high spatial-resolution data and/or in tandem with tissue segmentation, and is a subject for an on-going investigation. Estimation of partial derivatives tends to be rather sensitive to noise and other perturbations present in the B1 maps. Improvement of the robustness of the estimation by leveraging high field MR and customized data acquisition is also being investigated.

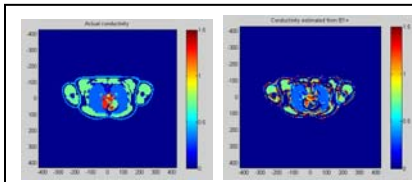

Fig. 1. Conductivity (left), estimated from B1+ (right)

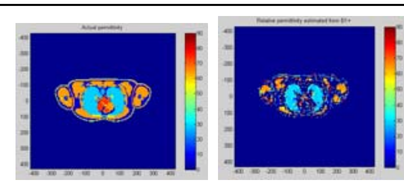

Fig. 2. Relative permittivity (left), estimated from B1+ (right)

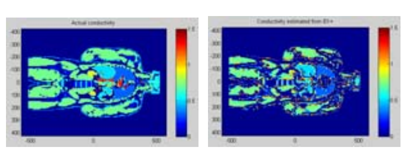

Fig. 3. Conductivity (left), estimated from B1+ (right)

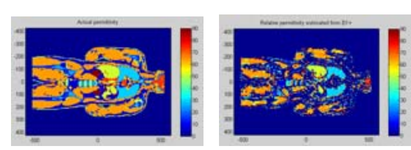

Fig. 4. Relative permittivity (left), estimated from B1+ (right)

The relative permittivity estimated from B_1^+ is shown in Fig. 2. Again, the equations accurately estimate the relative permittivity in regions of constant electrical properties with sufficient B_1^+ samples.

Corresponding results for central coronal plane are shown in Fig. 3 and Fig. 4.

Conclusions: Tissue conductivity and relative permittivity can be directly estimated from complex B_1^+ maps in a volume of constant electrical properties.

References: [1] Fear, et. al., IEEE Microwave Mag., Mar 2002 [2] Katscher, et. al., ISMRM 2006 [3] Nachman, et. al., ISMRM 2007 (Unsolved problems and unmet needs)

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