

# A Closed-Form Expression for Ultimate Intrinsic Signal-to-Noise Ratio in MRI

E. Kopanoglu<sup>1</sup>, V. B. Erturk<sup>1</sup>, and E. Atalar<sup>1</sup>

<sup>1</sup>Electrical and Electronics Engineering, Bilkent University, Ankara, Turkey

## INTRODUCTION

In magnetic resonance imaging (MRI), an increase in signal to noise ratio (SNR) provides higher quality images, making diagnosis easier, thus it should be maximized. The main noise sources in an MRI experiment are classified as preamplifier, coil and body noises. Preamplifier noise is small when ultra-low-noise amplifiers (ULNA) are utilized. Coil noise is also small in most applications. It becomes dominant when low-field imaging and/or small coils are used. In these cases superconductor or low temperature wires are used to minimize its effect. In most applications, body noise is the dominant factor in determination of the SNR of the images.

SNR of the images depends on many factors. Intrinsic SNR (ISNR) was defined as a measure of coil performance by removing imaging parameter dependent components from SNR; and it only depends on the coil structure and the electromagnetic and geometric properties of the body of interest. Therefore, the lowest upper bound on ISNR, which is called ultimate-ISNR (UISNR) [1, 2] provides a solid reference for coil performance evaluation. However, previous studies do not provide a closed-form expression and hence, the effects of the sample related parameters including size, conductivity, permittivity and permeability on the SNR are not clear. If the ultimate value of the ISNR is known in the closed-form, it can be easily calculated. With apriori information about the maximum ISNR that the coil can have, one can design RF coils, evaluate the performance of an existing coil and find out how much room for improvement exists. Furthermore, resolution and imaging time limits of an experiment can be understood easily, at the stage of experiment design.

With this motivation, we obtained a closed-form expression for UISNR for imaging cases for which wavelength is larger than the subject of interest, including small subject imaging, MR microscopy, high-field NMR spectroscopy and low field MRI.

## THEORY

In this paper, a conducting uniform cylindrical sample is assumed. The fields inside the sample are derived using cylindrical Bessel functions, which are the natural basis functions of the structure. The SNR is lowest in the center of the structure, due to the fact that external coils are farther to the center and coil sensitivity drops as distance increases. Therefore, we selected our point of interest as the center, to find the lowest upper bound on SNR which is given by

$$\text{UISNR}_{\text{ref}} = \frac{\omega \mu_0 M_0}{\sqrt{4\sigma k_B T}} \sqrt{\sum_{n=-\infty}^{\infty} \frac{(Q_{-2} + Q_0) \left[ |\beta^2|^2 + \left( \frac{2\pi n}{L} \right)^4 \right] + (Q_{-2} - Q_0) \left[ 2 \left( \frac{2\pi n}{L} \right)^2 \Re \{ \beta^{2*} \} \right] + 2Q_{-1} \left| \frac{2\pi n \beta_p}{L} \right|^2}{L (\omega \mu |\beta_p|)^2 (Q_{-1} Q_{-2} + Q_{-1} Q_0 + 2Q_{-2} Q_0)}} \quad (1)$$

where  $Q_m$  is the surface integral of  $|J_m(\beta_p \rho)|^2$ .  $\beta$  is the wavenumber and  $\beta_p = \sqrt{\beta^2 - (2\pi n/L)^2}$  is the radial propagation constant.  $\mu$  is the permeability,  $\omega$  is the Larmor frequency,  $L$  is the length of the structure,  $R_0$  is the radius of the structure,  $M_0$  is the equilibrium magnetization and  $n$  is the longitudinal mode number. The infinite summation can be replaced with a finite one using the fact that the contributions of higher order modes are less significant. Using infinite summation expansion of Bessel functions together with the assumptions that  $L > 4R_0$  and the sample being much smaller than the wavelength, a simple closed-form UISNR expression can be found as:

$$\text{UISNR}_{\text{closed-form}} = \frac{g}{\sqrt{k_B T}} M_0 \sigma^{-0.5} R_0^{-2.5} \quad (2)$$

where  $g=0.732$  is the geometry factor for a cylinder.

## DISCUSSIONS AND CONCLUSION

From (2), it can be seen that when the aforementioned assumptions hold, the ultimate intrinsic SNR is independent of material permittivity, permeability and length. Furthermore, the UISNR changes linearly with  $M_0$  and hence  $B_0$ . It was previously shown that SNR is inversely proportional with the 2.5<sup>th</sup> power of the radius [3]. Our study, which incorporates the concept of rotational magnetic field and gives the ultimate value of the ISNR also shows the same dependence on radius and hence does not conflict with [3].

As human/animal bodies are neither exactly cylindrical nor uniform, the derived relation has some intrinsic errors, which can be as high as 25%. With a smaller error margin of 20%, a uniform conductivity of 0.4 S/m and a relative permittivity of 80, (2) can be used for samples with a radius of 14 cm up to 1.5T which includes human imaging at low and intermediate field strengths. At 4.7T, 9.4T and 16T, (2) has 10% error at 5cm, 2cm and 0.8cm respectively. Hence, it can be used for MR microscopy, small animal imaging and high field NMR spectroscopy. On the other hand, when coil performances are to be found by imaging phantoms, the closed-form relation is very accurate. In the region of interest where the samples are smaller than the wavelength, the error remains less than 0.04%. The figure shows the accuracy of the closed-form solution.

The closed-form expression suggests linear change with the main magnet strength. Hence, the error should be a constant line. However, when the sample is comparable to the wavelength, the figure shows a non-linear decrease which indicates that the exact SNR solution has a higher order dependence on the main magnet strength and as a result, the closed-form expression under-estimates. This is a result that agrees with the previously published arguments [1, 2]. However, it should be noted that, the conductivity and permittivity of the samples are assumed to be constant whereas in real life, these properties are functions of frequency. Hence, the error outside the region of interest, where the samples are comparable with the wavelength, may not be exact.

To the best of our knowledge, this is the first study that derives a closed-form expression and shows the asymptotic behavior of UISNR for samples smaller than the wavelength. Also, the dependence on the conductivity of the sample is shown for the first time. Equation (2) can be used together with the imaging and preamplifier parameters as given in [4] so that one can easily calculate the SNR without the need of knowledge on computational implementation or UISNR theory. Also the resolution and imaging time limits of an experiment can be understood at the stage of experiment design.

## REFERENCES

- [1] O. Ocali, E. Atalar, Ultimate Intrinsic Signal to Noise Ratio in MRI, Magnetic Resonance in Medicine, 1998
- [2] H. Çelik, Y. Eryaman, A. Altıntaş, I.A. Abdel-Hafez, E. Atalar, Evaluation of Internal Coils Using Ultimate Intrinsic SNR, Magnetic Resonance in Medicine, 2004
- [3] D.I. Hoult, P. C. Lauterbur, The Sensitivity of the Zeugmatographic Experiment Involving Human Samples, Journal of Magnetic Resonance, 1979
- [4] W.A. Edelstein, G.H. Glover, C.J. Hardy, R.W. Redington, The Intrinsic Signal-to-Noise Ratio in NMR Imaging, Magnetic Resonance in Medicine, 1986.

