Diffeomorphic normalization via constant acceleration field

Y-C. Hsu[1], C-H. Hsu[1], and W-Y. I. Tseng[2]

[1]Department of Biomedical Engineering and Environmental Sciences, National Tsing Hua University, Hsinchu, Taiwan, [2]Center for Optoelectronic Biomedicine and Department of Medical Imaging, National Taiwan University College of Medicine and Hospital, Taipei, Taiwan

Introduction: Image registration plays an important role in neuroimaging research which requires between-subject analysis or within-subject time-dependent studies such as brain structure changes during development, disease progression or treatment. The general scenario of image registration is to 'normalize' the target image to the template image (or another image) in terms of minimizing certain cost functions. One of the simplest ways to perform normalization is to use affine transform; however, as many other high-dimension non-rigid methods, it is a small-deformation approach. This approximation is usually not proper since images vary greatly between subjects, and so image registration is usually a large-deformation problem. Under the diffeomorphic framework, Beg et al. proposed an LDDMM algorithm [1] which models image registration problem similar to fluid mechanics where target image 'flows' to match the template image. This method iteratively calculates the velocity field at each 'time point' such that an energy function is minimized. What the LDDMM generated is a minimum path between target and template images; however, it is time consuming because of its variable velocity fields. Ashburner proposed a fast algorithm [2] which simplified Beg's LDDMM method by replacing the variable velocity fields to a single one. Although Ashburner's method uses single constant velocity field to encode the whole path of transformation and consequently is not a minimum path method, it still belongs to diffeomorphic transformation. In this study, we extended Ashburner's concept by using a constant acceleration filed to encode the variable velocity fields.

Theory: $\phi_t(x)$ represents the location at time t of a particle originates at x at time 0. From fluid mechanics, we have

$$\frac{d}{dt}\phi_t(x) = v_t \circ \phi_t(x), \quad (1)$$

where $v_t(x)$ is the velocity field at time t. We further define the acceleration field $a_t(x)$ by

$$\frac{d^2}{dt^2}\phi_t = a \circ \phi_t = \frac{d}{dt}(v_t \circ \phi_t). \quad (2)$$

Consequently,

$$\frac{d}{dt}(v_t \circ \phi_t) = (Dv_t)\circ\phi_t \cdot \frac{d}{dt}\phi_t = (Dv_t)\circ\phi_t \cdot v_t \circ \phi_t. \quad (3)$$

Eq. (3) reduces to $a = (Dv_t)\cdot v_t$. Take partial derivative of a with respective to v_t yields $\partial a/\partial v_t = Dv_t$. Therefore the derivative of E_t with respective to a is

$$\frac{\partial E_t}{\partial a} = \sum_t \frac{\partial v_t}{\partial a}\frac{\partial E_t}{\partial v_t} = \sum_t (Dv_t)^{-1}\frac{\partial E_t}{\partial v_t}. \quad (4)$$

Materials and methods: Two T1-weighted images of matrix size 64×64×40 were used to demonstrate the presented method. Beg's LDDMM algorithm and Ashburner's constant velocity filed method were also implemented for comparison. For optimization, steepest gradient method was adopted for all methods. The 'flow' between two images was digitized into 20 steps in all methods. The path lengths ($\Sigma|v_t|$), and the energies ($\|I_0 \circ \phi_1^{-1} - I_1\|_{L_2}^2$) of the three methods were compared.

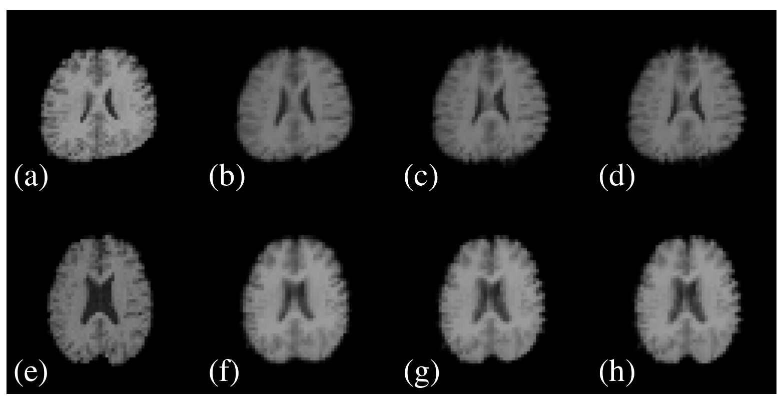

Fig. 1

Results: One selected slice is shown in Fig. 1, (a) and (e) are the original images, others are images transformed by our constant acceleration field method ((b) and (f)), Beg's LDDMM method ((c) and (g)) and Ashburner's constant velocity filed method ((d) and (h)), respectively. The transformed images in the upper row ((b), (c) and (d)) which are close to image (a) are derived from image (e), Similarly, images in the lower row ((f), (g) and (h)) which are close to image (e) are derived from image (a). Path lengths of the proposed method, the LDDMM algorithm and the constant velocity field method were 1.19, 1.02 and 1.13, respectively. Energies were 7.68 for our method, 7.55 for the LDDMM algorithm and 7.88 for the constant velocity field method.

Discussion: Our results showed that the proposed method – constant acceleration filed – diffeomorphically normalize one image to the other, so as the other two methods. Theoretically, the LDDMM method could give the results with the 'shortest path', whereas the other two methods, designed to encode the whole particle trajectory with a single velocity or a single acceleration field, may give a circuitous path [2]. Although LDDMM method gives shortest path length and lowest energy, the necessity of storing all velocity fields may become a problem when huge amount of image data is to be handled. Comparing our method and the constant velocity field method, the prior has lower energy and the later has shorter path length. The result suggests that these three methods might have their own strength and weakness in accuracy and computation cost.

References: [1] Beg, M.F., et al. Int. J. Comput. Vis. 2005;61:139–157. [2] Ashburner J, Neuroimage 2007 ;38:95– 113.