Data Interpolation in Phase-scrambling Fourier Transform Technique for Alias-free Image Reconstruction by Iterative Signal Restoration Algorithm

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Introduction

The phase scrambling Fourier transform (PSFT)[1] signal can be converted into the Fresnel transform[2] signal by multiplying by a quadratic phase term. Because the amplitudes of the adjacent samples of the Fresnel transform signal are highly interrelated, the signal amplitude at a point between sampled points can be estimated with a high degree of accuracy even if the sampling is coarse and aliases are generated in the reconstructed images. Since the coarsely sampled Fresnel transform signal does not satisfy the Nyquist sampling condition, however, a small residual alias is contained in the reconstructed image. In this paper, a new alias-free image reconstruction technique in the PSFT imaging technique is presented in which the signal is converted into the Fresnel transform signal and the residual alias in the image is removed by increasing the data interpolation accuracy with using iterative signal restoration algorithm. Numerical simulation using MR images showed that almost alias-free images can be reconstructed using this technique.

Data Interpolation in Fresnel transform Technique using Iterative Signal Restoration Algorithm

The signal in the PSFT technique which uses a quadratic nonlinear field gradient pulse in syncronization with the phase encoding gradient pulse in the conventional Fourier imaging technique is given by Eq.(1).

$$v(\gamma g_x t_x, \gamma g_y t_y) = \int_{-\infty}^{\infty}\int \left\{\rho(x,y)e^{-j\gamma b\tau(x^2+y^2)}\right\}e^{-j(\gamma g_x t_x x+\gamma g_y t_y y)}dxdy \qquad (1)$$

where $\rho(x,y)$ is the spin density in the x-y plane at the slice selected z-location, γ is the magnetogyric ratio, b is the coefficient of the quadratic nonlinear field gradient, τ is its impressing time, g_x and g_y are the read out and phase encoding gradients, respectively. The coarsely sampled PSFT signal is converted into the Fresnel transform signal by multiplying by the quadratic phase term, and the converted signal is the coarsely sampled Fresnel transform signal. The coarsely sampled Fresnel transform signal can be interpolated by using an iterative signal restoration algorithm described in following part. The Fresnel transformation can be implemented using two algorithms, and we use the method 2 Fresnel transform in the next iterative signal restoration algorithm.

i) The coarsely sampled PSFT signal (N x $N/2$) is converted into the Fresnel transform signal by multiplying by a quadratic phase term and reconstruct image by method 2 Fresnel transform algorithm after interpolating the Fresnel signal in the coarsely sampled direction, phase encoding direction, using an interpolation polynomial. Then fill zero in the outside region of the image in size of ($4N$ x $4N$). The image has a residual alias in the phase encoding direction.

ii) Set the data in the imaginary part of the image equal to zero to destroy the phase coherency of the alias. (Generally, the spin density image is assumed to be a real number.) Since the image in the outside of the base-band region (N x N) is the alias, set the outside of the base-band region equal to zero.

iii) Construct Fresnel signal in size of $4N$ x $4N$ from the image obtained in step ii) by taking method 2 Fresnel transform, and replace the signal in the base band region in size of N x N by the original coarsely sampled Fresnel signal, which is converted from the coarsely sampled PSFT signal.

iv) Reconstruct image in size of $4N$ x $4N$ by taking method 2 inverse Fresnel transform.

Then steps ii) to iv) are iterated many times to converge the process. Finally, the base-band region in size of N x N is extracted from the $4N$ x $4N$ Fresnel transform data in step iii). The extracted N x N data is the interpolated one of the coarsely sampled Fresnel signal. An almost alias-free image can be reconstructed using method 1 Fresnel transformation.

Simulation

Numerical simulation was performed using MR images acquired in conventional MRI. PSFT signals at a coarse sampling condition in size of 256 x 128 were generated numerically by computer calculation and were converted into Fresnel signals by multiplying by the quadratic phase term. Then the iterative signal restoration algorithm was applied. Figure 1(a) shows an example of converted coarsely sampled Fresnel signal from coarsely sampled PSFT signal and (b) is the reconstructed image in step i). Figure 2(a) shows the restored Fresnel signal after 30 times iteration, and (b) is the reconstructed image. Aliasing artifact is not seen in the image.

Discussion

Two conditions that setting the imaginary part of the image and outside of the base-band region equal to zero in step ii) and replace the signal in the base band region by the original coarsely sampled Fresnel signal in step iii) are necessary to be converged the iteration process and to obtain the restored Fresnel signal. This work demonstrates that alias-free image can be reconstructed from a coarsely sampled PSFT signal by using the described iteration algorithm. This demonstrates the feasibility of a new fast imaging method differed from the conventional parallel imaging. Advantage of this approach is that only a single signal receiving system is needed to implement the alias-free image acquisition.

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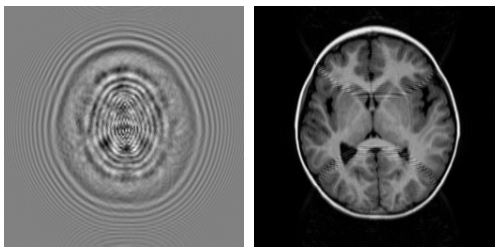
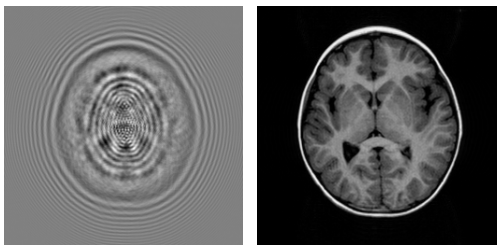


(a) **(b)**

Fig.1 (a): Fresnel signal converted from coarsely sampled PSFT signal in row direction (phase encoding direction) and interpolated by using polynomial expression, (b): Reconstructed image from (a). Alias is produced in phase encoding direction.



(a) **(b)**

Fig. 2 (a): Restored Fresnel transform signal after 30 times iteration, (b):Reconstructed image. Aliasing artifact is not seen in the image.