

# Bayesian reconstruction of DW-PROPELLER images using joint entropy

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## INTRODUCTION:

Diffusion-weighted MRI (DWI) provides a non-invasive method for in vivo evaluation of tissue water mobility. Most DWI studies employ single-shot EPI techniques which can suffer severe artifacts and image distortion in some applications. Multi-shot FSE-based DW-PROPELLER (1) and TurboProp (2) have shown to overcome these limitations. However, the scan time is usually longer to achieve a good SNR. Although the speed can be improved by reducing the number of blades, noise will become an issue. In this abstract, we propose a novel Bayesian reconstruction method to reduce the number of blades without compromising the SNR. The method incorporates the information from high quality non-weighted images into the DWI reconstruction, with the joint entropy between the weighted and non-weighted image features as the prior. A non-parametric model is used for the joint probability density function, which is estimated from the two images. The method is shown to improve the DW image quality with a set of DW-PROPELLER brain data.

## THEORY AND METHOD:

DWI images exhibit strong correlations with the non-weighted images. When used in reconstruction, the correlations can significantly improve the image quality of DWI (3,4). However, the relationship is complex and indirect due to different contrasts. We propose to use joint entropy as a metric for the correlation, which has been validated as a correlation metric for PET and MRI features (5). Bayesian method (6) is employed to incorporate the correlation in reconstructing the DWI images. Specifically, the desired DW image  $v$  is reconstructed by the maximum a posteriori (MAP) estimation  $v = \arg \max_v p(m|v)p(v)$ , where  $p(m|v)$  is the Gaussian likelihood function representing the measurement noise in  $m$ , and  $p(v)$  is the prior. We use a non-parametric model for the prior with a Gibbs distribution of the form  $p(v) = \alpha \exp(-\beta H(\mathbf{X}, \mathbf{Y}))$ , where  $\alpha$  is a normalization factor,  $\beta$  is a positive constant, and  $H(\mathbf{X}, \mathbf{Y})$  is the joint entropy of the random feature vectors  $\mathbf{X}$  and  $\mathbf{Y}$  from the weighted and non-weighted images. Only image intensity is considered as the feature in this work. The intensities in both images should follow similar distributions, though the actual values are not similar. Let the feature vectors extracted from both images be represented by  $x_i$  and  $y_i$ , which can be regarded as realizations of the random feature vectors  $\mathbf{X}$  and  $\mathbf{Y}$ . The joint entropy of two images is defined as  $H(\mathbf{X}, \mathbf{Y}) = -\sum_{x,y} p_{x,y} \ln p_{x,y}$ , where  $p_{x,y}$  is the joint density of  $\mathbf{X}$  and  $\mathbf{Y}$ . Using a non-

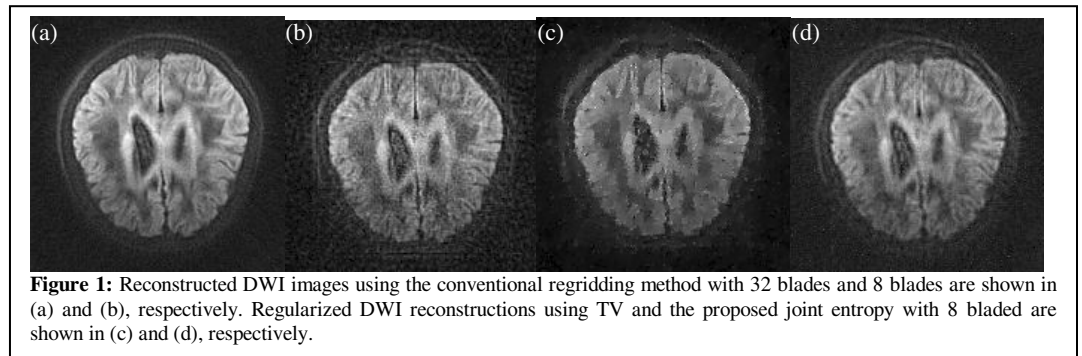
parametric approach, this joint density is estimated from  $x_i$  and  $y_i$ , which are the image intensities of the weighted and non-weighted images

respectively, using Parzen windows (7) in the form of:  $p(\mathbf{X}, \mathbf{Y}) = \sum_{i=1}^N \phi(\frac{\mathbf{X} - x_i}{\sigma_x}) \phi(\frac{\mathbf{Y} - y_i}{\sigma_y})$ , where  $\phi$  is a Gaussian window and  $\sigma$  determines the width

of the window. The window width is taken as a design parameter. Under this model, the reconstructed image by the MAP estimation is given as  $v = \arg \min_v (\|m - Ev\|_2^2 + \beta H(\mathbf{X}, \mathbf{Y}))$ . The above optimization problem is solved by the iterative nonlinear conjugate gradient algorithm (8). To speed up the computation, nonuniform fast Fourier transform using min-max interpolation (NUFFT) (9) was employed for the regridding of the non-Cartesian trajectory.

## RESULTS:

Brain images of a healthy volunteer were acquired from a 3T commercial scanner (GE Healthcare, Waukesha, WI) with an 8-channel brain coil ( $192 \times 192$  matrix) using a TurboProp sequence. A total of 32 blades were acquired and used to generate a high SNR DWI image (Fig. 1(a)) for comparison. To test different reconstruction algorithms, only 8 blades (each with 30 lines) were used. The reconstructions using the conventional regridding method, the



**Figure 1:** Reconstructed DWI images using the conventional regridding method with 32 blades and 8 blades are shown in (a) and (b), respectively. Regularized DWI reconstructions using TV and the proposed joint entropy with 8 blades are shown in (c) and (d), respectively.

total variation (TV) regularization and the proposed method and are shown in Fig. 1(b), (c), and (d), respectively. All the algorithms were implemented in MATLAB (MathWorks, Natick, WA). The proposed method is seen to be able to suppress the noise in the conventional regridding method and maintain the details that are lost in TV reconstruction.

## CONCLUSION AND DISCUSSION:

A novel method is proposed to improve the DWI image quality using its correlation with the high quality non-weighted image. The results show that the proposed method effectively suppress noise without reducing resolution like TV regularization does. This is because the joint entropy is computed from the (joint) grey level histogram and operates on individual pixel intensities. Future work will consider additional features such as the horizontal and vertical gradients at each pixel because the boundaries in the two images should be similar, and local mean in a neighborhood because the intensities should follow similar homogeneous distributions within the boundaries.

## REFERENCES:

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