Extended Anti-Aliasing Reconstruction for Phase-Scrambled MRI with Quadratic Phase Modulation

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Introduction

Phase-scrambled (PS) MRI is capable of a scalable image reconstruction with interesting anti-aliasing options [1,2]. The purpose of the present work is to extend the concept via a transparent mathematical framework capable of handling scalable alias-suppressing image reconstruction based on acquired PS-MRI signals.

Theory

Consider without loss of generality MRI signal of a sample $\rho(x)$ in a 1D case:

$$s(k)=\int \rho(x)\exp(-i2\pi kx)\,dx=\int \rho(x)\exp\left(-i2\pi\frac{k}{k_{\max}}\frac{x}{\Delta x}\right)dx. \qquad (1)$$

With a trivial substitution $2ab=a^2+b^2-(a-b)^2$ Eq. 1 transforms into:

$$s(k)=\exp\left(-i\pi\left(\frac{k}{k_{\max}}\right)^2\right)\int \rho(x)\exp\left(-i\pi\left(\frac{x}{\Delta x}\right)^2\right)\exp\left(i\pi\left(\frac{k}{k_{\max}}-\frac{x}{\Delta x}\right)^2\right)dx. \qquad (2)$$

This expression can be simplified by defining $g(\vartheta)=\exp(i\pi\vartheta^2)$, and upon following substitutions $\eta=k/k_{\max}$, $\xi=x/\Delta x$, $\rho'(\xi)=\rho(\xi\Delta x)\Delta x g^*(\xi)$, $s'(\eta)=s(\eta k_{\max})g(\eta)$, as:

$$s'(\eta)=\int \rho'(\xi)\exp\left(i\pi(\eta-\xi)^2\right)d\xi, \text{ or } s'=\rho'\otimes g. \qquad (3)$$

Eq. 3 states that the Fourier transform of the modified spin density $\rho'(\xi)$ can be expressed in terms of a convolution with a Gaussian phase or a *chirp* function, and defines a basis of the proposed analytic framework. Interestingly enough, as $g(\eta)$ is a pure phase term and its Fourier transform is a Gaussian phase multiplied by a constant, and bearing in mind, that division by a phase factor can be replaced with a multiplication by a complex conjugate, image reconstruction can also be formulated in terms of a convolution:

$$\rho'=s'\otimes g^*. \qquad (4)$$

If in the above equations a modified chirp function $g_\alpha(\vartheta)=\exp(i\pi\alpha\vartheta^2)$ is used, an image scaled by a factor α will result, in concert with a PS-MRI proposed reconstruction [1]. It is to be noted, that this is equivalent to a scaled MRI reconstruction based on a Chirp-Z transform [3]. Note, that neither the continuous integration presented here nor the approach as in [1] has yet provided insights for anti-aliasing abilities of the proposed reconstruction.

Methods

Simulations were performed in Matlab (The Mathworks, USA) using a Shepp-Logan head phantom with modified intensities. MRI scans were performed on a Siemens TIM-Trio 3T system (Siemens Healthcare, Erlangen, Germany) in a healthy volunteer.

Results and Discussion

Aliasing can be appreciated by discretizing Eq. 4 and observing the properties of the chirp function g_α. As seen from Fig. 1a for $\alpha=1$ modulation reaches the Nyquist limit at the edge of FOV, correspondingly the phase increment between the neighbouring pixels approaches π. For $\alpha=2$ aliasing of the discrete representation of g_α occurs (Fig 1c,d) and the convolution kernel be becomes periodic, resulting in aliasing in the reconstructed images. To suppress aliasing it is necessary to modify the convolution kernel in Eq. 4.

In [1] Eq. 4 is solved in Fourier domain via multiplication of $FT(s')$ with the analytically calculated function G_α, a Fouier image of g^*_α. Although G_α exhibits no aliasing artefacts for $\alpha \geq 1$, its discrete Fourier transform (DFT) shows a non-trivial behaviour. As seen from Fig. 2, presenting the ratio of $DFT(G_\alpha)$ to g^*_α (blue and red), $DFT(G_\alpha)$ is windowed at the position where g^*_α achieves the Nyquist limit. We propose to constrain explicitly the convolution kernel with a suitable window, for example based on a Fermi band-pass filter (Fig.2, green line), with an advantage of a flexible control of ringing and aliasing artefacts.

Convolution reconstruction and associated requirements are visualised in Fig. 3. As seen from Fig. 4, increase in quadratic phase variation causes k-space signals relocation and its appearance to resemble an image. Phase modulation in the object causes displacements of the k-space echo positions, d_k, proportional to the local phase gradient. For quadratic phase modulation d_k is proportional to the distance to the modulation centre, l. In reality phase shall not strictly be quadratic, but scaling reconstruction with a kernel of width w is only possible when $d_k+w/2<l$. Phase modulation thus shall be predominantly quadratic and rather significant, with the strength of the induced modulation comparable to that of imaging gradients. If phase modulation is low, aliasing occurs (Fig.5, left); an excess modulation causes intravoxel dephasing at the periphery of the FOV (Fig.5, right).

Unaliasing of the undersampled in vivo gradient echo data is shown in Fig. 6. With the second order shim current set to the limit TE=25ms was required to achieve a sufficient phase modulation, which demonstrates a need for stronger switchable shim coils.

Our results show that translation of the Fresnel transform concept to k-space representation allows to define limits and boundary conditions for practical implementations and to understand signal behaviour and potential artefacts. Scalable image reconstruction may benefit multiple applications, where low resolution full FOV images reconstructed from partial data can be used for navigation or as intrinsic reference or calibration data.

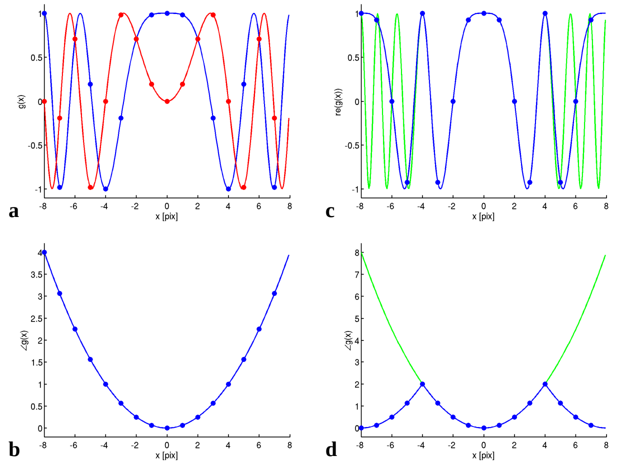

Fig. 1. *(a)* ***Real*** *and* ***imaginary*** *parts of the chirp kernel for $\alpha=1$ and (b) its phase. Phase increment reaches π at the edge of FOV. (c) Same for $\alpha=2$ in* ***continuous*** *depiction and* ***sampled*** *(d)* ***Analytic*** *phase of the chirp for $\alpha=2$ and its phase unwrapped from* ***discrete*** *samples.*

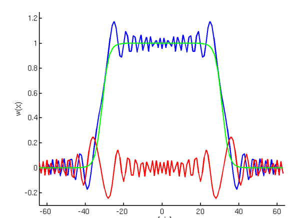

Fig. 2. *Window functions imposed on a generic chirp kernel for $\alpha=2$. Real (blue) and imaginary (red) parts of the window function implicitly used in [1] and a Fermi window (green) proposed here*

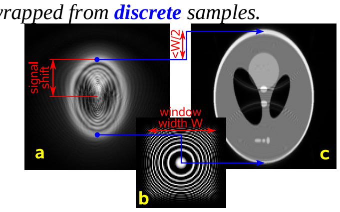

Fig.3. *Convolution reconstruction with a windowed kernel Quadratic phase modulation causes position-dependant shifts of MRI signals (a). Convolution kernel of width W (b) is able to transfer signals to their true positions in image space (c) if the signals in k-space matrix are within W/2 from their source in the image.*

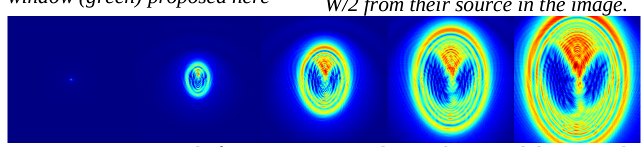

Fig. 4. *PS-MRI signals for increasing quadratic phase modulation in the range [0…3.8×10^{-6}]rad/pix^2. Quadratic phase within the object causes k-space signal shifts proportional to the position of the source (s.a. [2]).*

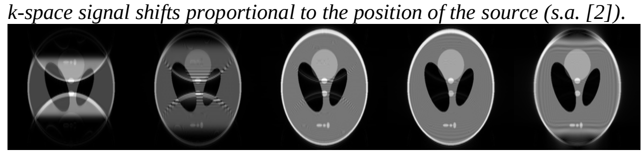

Fig. 5. *Images reconstructed from signals in Fig. 4 with $\alpha=2$ from half the k-space lines. As seen, insufficient phase modulation results in aliasing; Excess phase causes intravoxel dephasing at the periphery.*

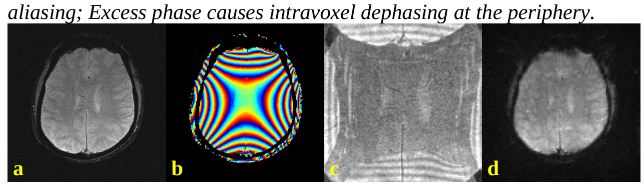

Fig. 6. *FOV unwraping in vivo. (a) Mgnitude and (b) phase reconstructed from full data 256^2 samples. (c) Wraped image and (d) convolution reconstruction of the retrospectively undersampled data (128^2 matrix)*

References: [1] Ito, MRM-08 60:422-30. [2] Pipe, MRM-95 33:24-33. [3] Kaffanke, JMR-06 178:121-8. **Acknowledgement:** BMBF Grant #13N9208