Kaczmarz Iterative Reconstruction for Arbitrary Hybrid Encoding Functions

J. P. Stockmann[1], and R. T. Constable[2]

[1]Biomedical Engineering, Yale University, New Haven, CT, United States, [2]Diagnostic Radiology & Neurosurgery, Yale School of Medicine, and Biomedical Engineering, Yale University, New Haven, CT, United States

INTRODUCTION: The Kaczmarz iterative algorithm – also known as ART, or Algebraic Reconstruction Technique – is commonly used to reconstruct images from projections in CT, cryo-EM, and other modalities [1]. Owing to the efficiency and ease of reconstructing phase-encoded k-space MR data via the 2D Fourier Transform, the Kaczmarz algorithm has not found widespread application in MR imaging. However, with the advent of imaging methods employing higher-order gradient shapes which do not generate k-space data in the conventional sense [2]-[3], a reconstruction method is needed which can efficiently handle arbitrary encoding functions. Advantages of the Kaczmarz algorithm include the ease of implementation and its ability to work with any encoding function, i.e., any combination of coil profiles and gradient shapes.

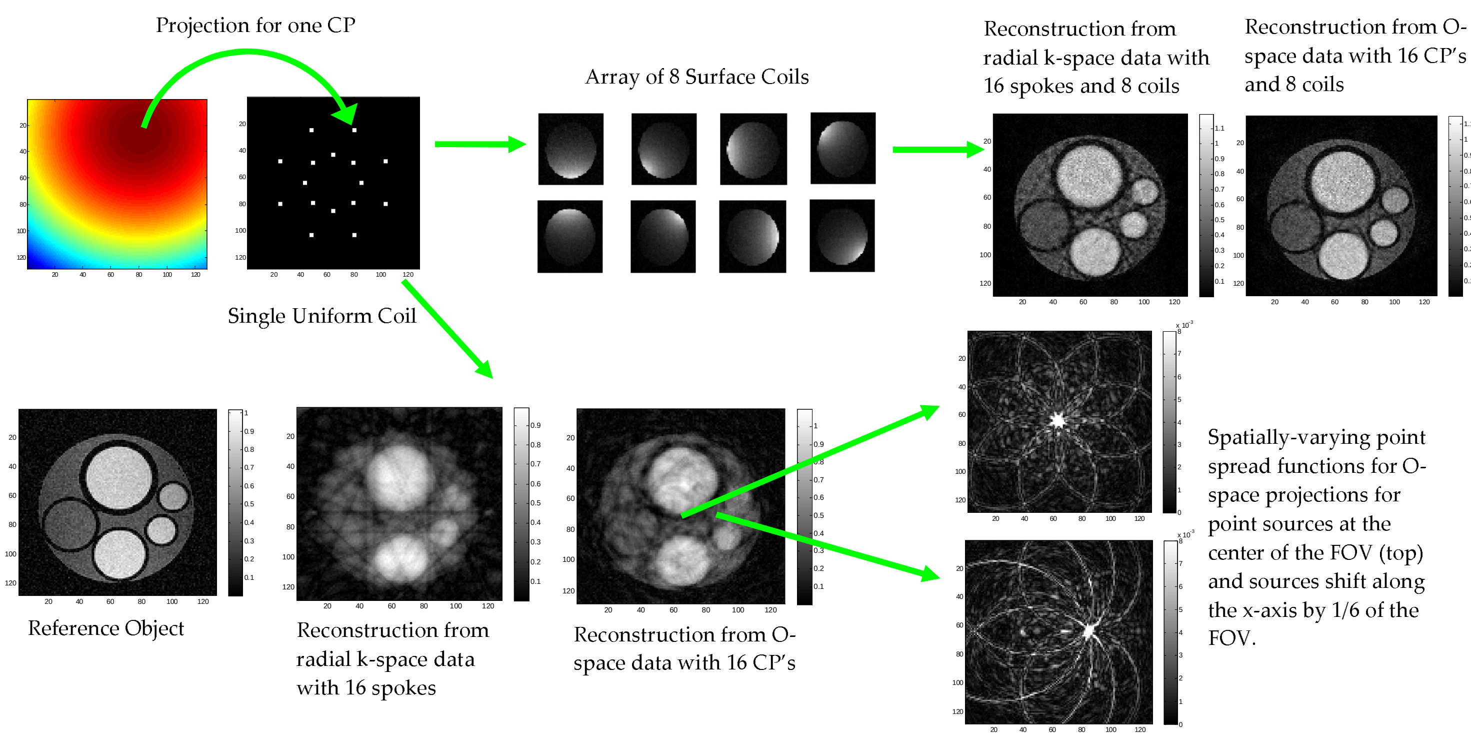
METHODS: We use the Kaczmarz algorithm to reconstruct "O-space" projections generated using off-center quadratic gradient functions corresponding to a combination of first and second-order spherical harmonics (X,Y, and Z2) along with a B0 offset. Projections generated with such an encoding function can not be handled using a k-space density compensation and re-gridding approach because the encoding function is not in the form of a Fourier integral kernel, so the data do not reside in k-space.

$$\underbrace{s_{m,n}(t)}_{\text{Echo at time } t \text{ produced by } n^{th} \text{ coil and } m^{th} \text{ CP}} = \iint \overbrace{\rho(x,y)}^{\text{Sample}}\underbrace{C_n(x,y)e^{-j2\pi G\left((x-x_m)^2+(y-y_m)^2\right)t}}_{\text{Hybrid encoding /basis function}}dxdy = \overbrace{A_{m,n,t}}^{\text{Projection Matrix}}\rho$$

The Kaczmarz algorithm directly solves the integral equation for the unknown sample, obviating the need to Fourier transform the echo. The integral kernel is represented as a projection matrix $A_{m,n,t}$ for each time point t for the n^{th} coil and m^{th} center placement (CP). The algorithm compares each data point with the inner product of the projection matrix with the n^{th} iterate of the estimator. This difference weights the amount of the basis function in $A_{m,n,t}$ which is added to the estimator going into the next iteration.

$$\hat{\rho}_{n+1} = \hat{\rho}_n + \frac{s_{m,t} - \left\langle A_{m,t}, \hat{\rho}_n \right\rangle}{\left\| A_{m,t} \right\|^2} A^*_{m,t}$$

The algorithm typically converges in just a few iterations [1]. The entire projection matrix – spanning all time points, coils, and center placements – is usually too large to fit in memory, but since only one data point is treated at a time, the matrix for each individual basis function can be stored to the hard drive and loaded into memory when necessary.



DISCUSSION: We demonstrate the versatility of the Kaczmarz algorithm for reconstructing 128x128 images from 8-fold undersampled datasets with and without coil encoding. While the algorithm is able to reconstruct non-Cartesian k-space data from linear gradients, we feel that its most powerful application is the reconstruction of projections formed using non-linear gradient shapes. However, when measured data are used, poor reconstructions result if the encoding matrix is not known precisely. Future efforts will focus on improving the algorithm's robustness in the presence of uncertainty as to the exact gradient strengths applied during an experiment.

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