Fast Phase Unwrapping Using Fourier Transform

A. M. Abduljalil[1], S. Choi[1], M. V. Knopp[1], and P. Schmalbrock[1]
[1]Radiology, The Ohio State University, Columbus, OH, United States

Introduction: The calculation of the phase of a complex MR image produces phase wraps when the phase changes exceed 360°. The main contribution to this wide range of the phase is the accumulated phase caused by a non-homogeneous magnetic field, as well as the susceptibility induced field gradient caused by the imaged object itself, which increases with the strength of the magnetic field. It is expected that if pixel-wise phase change is less than 360° then it is possible to unwrap the phase. The phase extraction using discrete Fourier transform method is explored.

Theory: In the image space, the phase of the complex image $\vartheta(r)$ as a function of position **r** can be expressed in the function $\psi(\boldsymbol{r}) = e^{i\vartheta(r)}$. The Fourier transform method [1] can be summarized in the following steps. First, by defining the forward and backward Fourier transforms of $\psi(\boldsymbol{r})$ as:

$$\mathcal{F}\,[\vartheta(\boldsymbol{r})] = \int \vartheta(\boldsymbol{r})\, e^{-2\pi i\, \boldsymbol{k}.\boldsymbol{r}}\, d\boldsymbol{r} \qquad (1)$$

and $$\vartheta(r) = \int \mathcal{F}[\vartheta(r)]\, e^{2\pi i\, \boldsymbol{k}.\boldsymbol{r}}\, d\boldsymbol{k} \qquad (2)$$

The gradient of Eq. (2) is:
$\nabla\,\vartheta(r) = 2\pi\, i \int \mathcal{F}[\vartheta(r)]\, \boldsymbol{k}\, e^{2\pi\, i\, \boldsymbol{k}.\boldsymbol{r}}\, d\boldsymbol{k}$, which can be rearranged as: $\nabla\,\vartheta(\boldsymbol{r}) = 2\pi i\, \mathcal{F}^{-1}[\mathcal{F}[\vartheta(\boldsymbol{r})]\boldsymbol{k}]$ (3)
Hence, $\mathrm{F}[\nabla\vartheta(\boldsymbol{r})] = 2\pi i\, \mathcal{F}[\vartheta(\boldsymbol{r})]\, \boldsymbol{k}$, then, with the inverse transformation of this equation one can obtain:

$$\vartheta(\boldsymbol{r}) = \mathrm{Re}\left\{\frac{1}{2\pi i}\, \mathcal{F}^{-1}\left[\frac{\mathcal{F}[\nabla\vartheta(\boldsymbol{r})].\boldsymbol{k}}{k^2}\right]\right\} \qquad (4)$$

where the only unknown in Eq. (4) is $\nabla\vartheta$ which can be evaluated from the gradient of $\psi(\boldsymbol{r})$ that can be expressed as: $\nabla\vartheta(\boldsymbol{r}) = \nabla\psi(\boldsymbol{r})\,/\,i\,\psi(\boldsymbol{r})$.

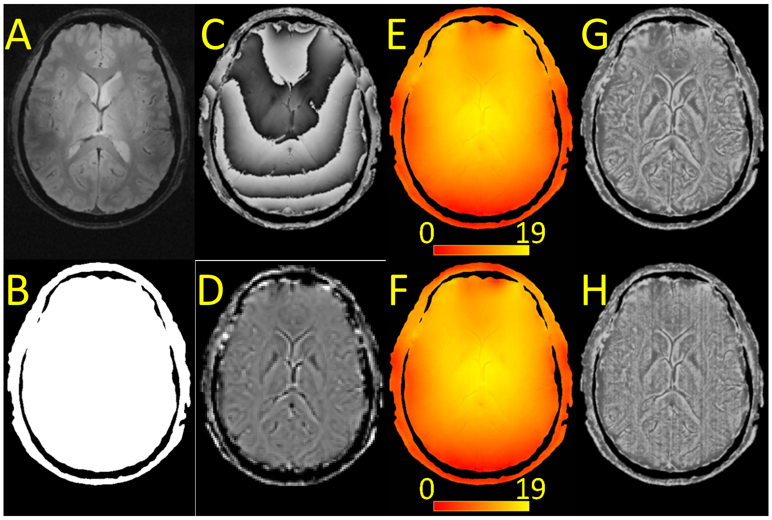

Figure 1 (A) Magnitude image, (B) image mask, (C) wrapped phase image calculated from arctangent of the complex image, (D) unwrapped and filtered phase, (E,F) unwrapped phase where the calculations were performed using masked and unmasked complex image respectively, the phase value is shown in radian, (G,H) the corresponding histogram equalized unwrapped and filtered phase.

Method: This method is tested with multi-slice and 3D acquisitions. Fig. 1 shows a slice from a 2D gradient echo data set acquired with a single transmit and receive coil with a matrix of 384x512, slice thickness 2.5 mm, TR/TE 1600/12 ms using a 7 T Philips Achieva system. Fig.1A shows the magnitude image and (B) an image mask which was generated using a simple threshold the magnitude image after the application of a smoothing filter. (C) Shows the wrapped phase. The phase was unwrapped using the above method from the complex phase in the image space. The phase was calculated from the masked and unmasked data and displayed in Fig. 1E and F. To remove the spatially slow varying phase change, a convolution filter was applied to the unwrapped phase. The difference is shown in (D). The applied convolution used a 31x31 kernel with a Gaussian shape. Histogram equalized phase images for masked and unmasked data is shown in (G) and (H) respectively.

Discussion: The applied unwrapping method is robust and fast, since it involves mainly the calculations of discrete Fourier transforms. Comparison between processed images with and without the use of a noise mask can be visualized easier in Fig. 1G and H where the use of the mask improved the phase contrast to noise ratio. The inclusion of the noise region produced ripples in the unwrapped phase with a standard deviation of 0.3 radians. The unwrapped phase is similar to that generated using the PRELUDE algorithm [2] and the method can be applied in multiple dimensions as well.

References:[1] Volkov V and Zhu Y, Optics Lett 28 (2003) 2156-2158. [2] Jenkinson M, Magn. Reson. Med. 49(2003) 193–197.