# Improved temporal resolution in radial k-space sampling using an hourglass filter

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**INTRODUCTION:** Radial sampling of k-space is known to simultaneously provide both high spatial and high temporal resolution. Recently, an optimal radial profile time order based on the Golden Ratio was presented in [1]. We have adopted and modified the idea, with a focus on higher temporal resolution without sacrificing any image quality.

**MATERIALS AND METHODS:** K-space was sampled radially using $N$ profiles, see Fig. 1, where $N$ = 7. The time order of the profiles was not implemented using a conventional angular increment of $\Delta\phi$ = 180°/$N$. Instead, the angular increment $\Delta\gamma$ of the profiles was chosen to be as close as possible to 180° divided by the **golden ratio**, $\Delta\gamma$ =180°/(√5+1)/2≈111°. If an arbitrary time interval was chosen, the profiles within that interval would be spread out almost evenly across k-space [1]. The number of profiles was chosen to be a **prime number**, which guaranteed that all profiles will be visited before repeating the first profile [2]. It can be shown that in the case of $N$ =7 profiles, the angular increment will be 103°. The time order t = 1, 2, 3, 4, 5, 6, 7, 1, 2, 3, 4, ... of the profiles is marked in Fig. 1.

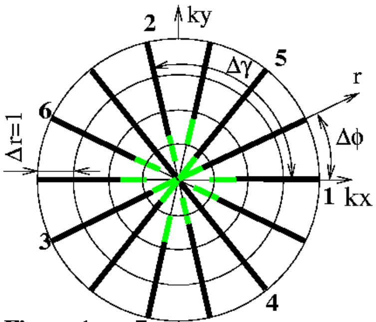



**Figure 1**

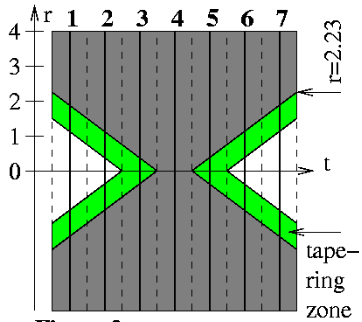
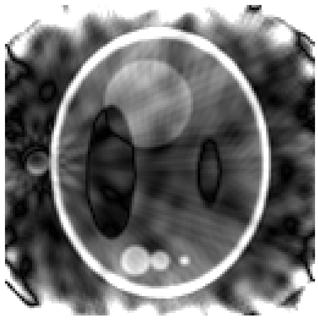



**Figure 2**

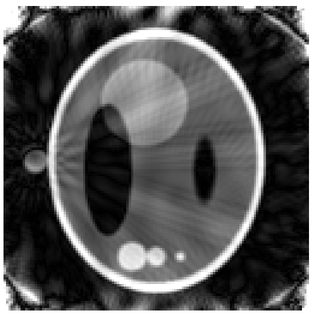
The **gridding** method [3], [4], was used for image reconstruction. The radially sampling pattern ($r$, $\phi$) is re-sampled to a Cartesian grid before (inverse) Fourier transform. The algorithm consists of the following steps: 1) Pre-compensation with the local sampling density, 2) Resampling with forward mapping using the interpolation function $C(k_x,k_y)= C(k_x)\cdot C(k_y)$ of data values and weight values, 3) Post-compensation with the weight function, 4) Inverse Fourier transform, 5) Division with the inverse Fourier transform of $C(k_x,k_y)$. A 4-point Kaiser-Bessel filter for $C(k_x)$ was used. The pre-compensation was proportional to $2\pi r/N$ except for the samples at the origin which was compensated using $0.5^2\pi/N$. The sampling distance of the Cartesian grid $\Delta x$ was chosen to the radial sampling distance $\Delta r$=1. To avoid under-sampling, the angular sampling density $r\pi/N$ should not exceed 1, i.e. $r \leq N/\pi$. In Fig. 2, the profiles were plotted with respect to time. For $N$=7, $r$=7/$\pi$ =2.23, is also indicated in the figure. For a radius larger than 2.23, aliasing will appear as streaks in the image. However, a small portion of aliasing can be tolerated, in particular as a smaller number of profiles will provide a better time resolution. On the other hand, for $r$=0 only one profile is required. If these points are connected, an **hourglass**-shaped contour is obtained. The points outside the hourglass are not needed and can therefore be removed thereby providing much better time resolution. According to [3], 'post-compensation' alone works well if the rate of change of the sampling pattern is not too rapid. However, in our application there are rapid changes in sampling density where the hourglass cuts the profiles. In addition, it is not straight-forward to perform a pre-compensation. Therefore, we introduced '**tapering zones**' in which the data was tapered off smoothly using a Gaussian function. The tapering weights were later included in the post-compensation.

**RESULTS:** Two properties were investigated in detail in the simulations, the temporal resolution and contingent artifacts introduced in the process. Fig.3A shows the reconstructed image using the conventional sliding window and Fig.3B-C show the influence of hourglass and tapering zones. The image size was 128×128, the number of profiles was $N$=61, and the intensities of the sub-objects in the large object were in the range [0.32-2.0]. The left outer object has intensity 3.0 and it was used as a sample object for evaluating the temporal resolution. Note that streak artifacts emanates from this object in all images. It was removed at $t$=63 and Fig.3A-C are shown at t=99. Fig.3D shows how the intensity of the removed sample object changes with time. The temporal resolution was defined as the transition time from 80% to 20% intensity. Note how the temporal resolution was increased by a factor of 1.7 when the hourglass filter and the tapering zones were used. At the same time the image quality was almost intact. The temporal resolution increased by a factor of 3 when hourglass was used without tapering, but image quality was highly degraded.

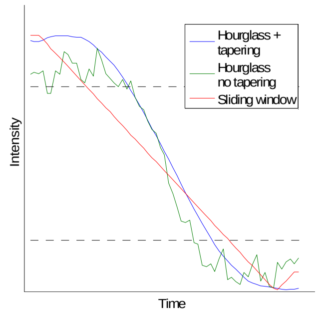
**Fig 3A:** Sliding window.



**B:** Hourglass without tapering.



**C:** Hourglass with tapering.




**D**: Time resolution for outer object.

**DISCUSSION:** The number of profiles $N$ clearly determined the angular resolution. A small portion of angular aliasing giving rise to fine streaks can be tolerated if the aim is much improved time resolution. A special feature was that $N$ was locked to be a prime number which guaranteed $N$ fixed angles. This resulted in avoidance of the moving streak artifacts. In contrast, this was not the case in [1] when the angular increment was based on an irrational number. The time resolution increased dramatically when the profiles were partly removed from the k-space using the hourglass filter. Rapid changes in k-space also degrade the image quality seriously, but is alleviated by the smooth tapering zone.

**REFERENCES:** [1] Winkelmann et. al.: *An optimal radial profile order based on the golden ratio for time-resolved MRI*, IEEE Trans. Med. Im, Vol.26, No.1, 2007. [2] Sunnegårdh: Combining *analytical and iterative reconstruction in helical cone-beam CT*, Linköping Studies in Science and Technology, Thesis No. 1301, 2007. [3] Pauly: *Non-Cartesian Reconstruction*, Lecture notes, URL: http://www.stanford.edu/class/ee369c/notes/non_cart_rec_07.pdf, 2007, [4] Thyr: *Method for Acquisition and reconstruction of non-Cartesian 3-D sampling,* Master Thesis LITH-ISY-EX--08/4058--SE, Linköping University, Sweden, 2008.

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