Reconstruction of Undersampled MR Data using Lp (0<p<1) Spatial Constraints

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Introduction: L1 norm constrained reconstruction/compressed sensing techniques [1, 2] have recently been proposed to accelerate data acquisitions in MRI by acquiring fewer data in k-space. Image artifacts due to k-space undersampling are resolved by minimizing L1 norm of the sparse image estimate while preserving fidelity to the acquired data. Using an L1 norm constraint exploits implicit sparsity in the data [3]. More recent developments to this approach include *(i)* using a priori information about the signal [4,5] *(ii)* re-weighting the signal [6] *(iii)* reordering the signal estimate [7,8] which enhance signal sparsity and improve the reconstructions. In a different direction, it has been suggested that an L*p* (0<*p*<1) constraint can obtain faithful reconstructions when undersampling is more severe [9]. Theory suggests that the minimum number of measurements required to obtain good reconstructions can be reduced as *p* is reduced [10]. In ref. [9] results were presented using simulations in which L0.5 constraint lead to significantly improved reconstructions over the corresponding L1 norm reconstructions. Here we test the feasibility of using L*p* constraint on MR Cartesian acquisitions and compare the results to those from the more standard L1 norm constraint.

Methods: Reconstruction from undersampled k-space data is performed by iteratively minimizing the cost function shown below:

$$\min_{\tilde{m}} \left\{ \left\| WF\tilde{m} - \tilde{d} \right\|_2^2 + \alpha \left\| \sqrt{\nabla_x \tilde{m}^2 + \nabla_y \tilde{m}^2 + \varepsilon} \right\|_p^p \right\} \quad (0 < p \le 1)$$

In the above expression, $\tilde{m}$ is the reconstructed image estimate, W is the binary undersampling pattern, F represents the Fourier transform, $\tilde{d}$ is the acquired undersampled k-space data. $\|.\|_p^p$ represents the L*p* constraint which is given by $\|m\|_p^p = \sum_{k=1}^{n} |m_k|^p$. As noted in [9], the above cost function is non-convex for *p*<1 due to the constraint term. However, using gradient descent minimization technique with a small step size and with a minimum L2 norm solution as the initial guess, improved reconstructions were obtained. The regularization parameter α was chosen optimally for *p*=1 case using the L-curve method [11] and the same value was used for other L*p* reconstructions. The reconstructions were tested on MR images of the pelvis, abdomen (1.5T) and brain (3T). Full k-space data were generated by computing the Fourier transform of the images and then undersampled by various reduction factors (R) in a Cartesian variable density random fashion [1] in which nine central phase encode lines were fully sampled while the outer regions were sampled in a pseudo-random fashion.

Results: Figure 1 shows the results of the reconstruction on an image of the pelvis for *p*=1 and *p*=0.5. The scan parameters for the pelvis were: TR=171 ms, TE=1.93 ms, FOV=285X214 mm^2, slice thickness=10mm, pixel size=0.74X0.74 mm^2. Ghosting artifacts along the phase encoding direction due to undersampling are lower in Fig. 1d (*p*=0.5) than in Fig. 1c (*p*=1) as shown by the difference images (Figs. 1e,1f). The root mean squared (RMS) error compared to the fully sampled data for *p*=0.5 was ~20% lower than that for *p*=1. Figure 2 shows a plot comparing the RMS errors for different reduction factors and for different L*p* reconstructions, demonstrating that lower values of *p* yield more accurate images. Similar trends were observed in the brain and abdomen data sets. Reconstruction times for p<1 were longer (up to ~3 times higher, ~15 min) than the L1 reconstruction time for the Matlab implementations.

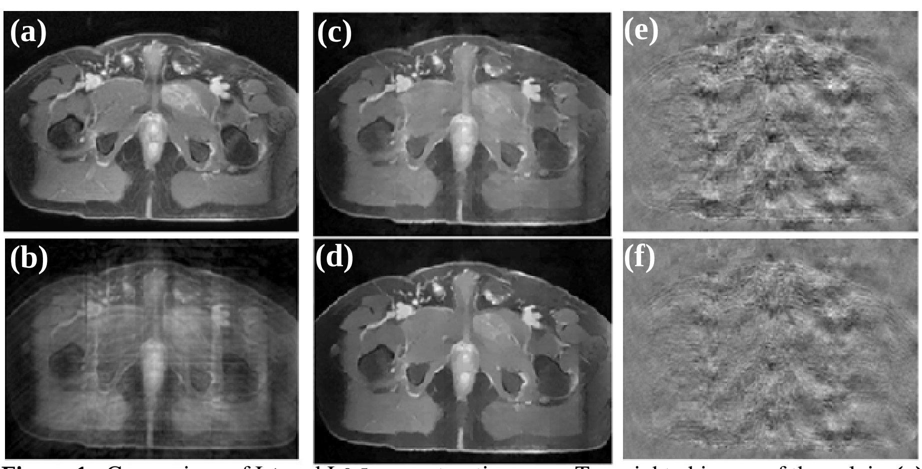

Figure 1. Comparison of L1 and L0.5 reconstructions on a T_1-weighted image of the pelvis. **(a)** Original fully sampled image of pelvis. **(b)** Undersampled reconstruction from R~3 Fourier data without any constraint. **(c)** L1 norm spatial constrained reconstruction from R~3 Fourier data. **(d)** Corresponding L0.5 constrained reconstruction. **(e)** Difference image between Figs 1(a) and 1(c). **(f)** Difference between 1(a) and 1(d).

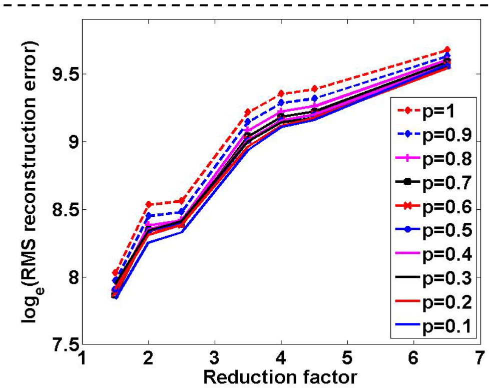

Figure 2. Comparison of reconstruction errors for different L*p* (0<*p*≤1) spatially constrained reconstructions and for different reduction factors for the image shown in Fig 1.

Discussion and Conclusion: Preliminary results suggest that L*p* spatially constrained reconstruction can offer improved reconstructions over direct L1 reconstructions. It was previously reported that using an L1 norm or total variation regularization leads to loss of contrast of small structures in the estimated images [1, 12]. L*p* reconstruction may be used to restore some of the loss of contrast. Although the cost function with an L*p* constraint is non-convex, iteratively minimizing the cost function until convergence yielded better image quality.

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