Compressed Sensing in Parallel Imaging: Towards Optimal Sampling

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Introduction

Compressed sensing (CS) is a novel method to measure and reconstruct N-dimensional compressible signals from $M << N$ linear-combination (*e.g.* Fourier-component) samples [1,2]. CS has been applied to brain MRI to achieve acceleration factors $R = N/M$ of 2–3 with only modest degradation in image quality [3]. A further reduction in the scan time can be achieved by multi-coil parallel MRI (pMRI) [4]. The two techniques have been recently combined to achieve high acceleration factors [5]. However, the performance of CS reconstruction relies on incoherent (*e.g.* random) sampling of the k-space. Here we show that a truly random k-space sampling in the context of pMRI allows an 8-12% reduction in the reconstruction error compared to the simple sampling patterns proposed earlier.

Methods

To fit the CS formulation an N_c-channel pMRI signal y of the unknown image x in the presence of noise e is formulated as $y = \mathbf{A}x + e$, where y and e are vectors of $N_d N_c$ data values; x is a vector of N pixels and N_d determines the level of under-sampling. $\mathbf{A}$ is an $N_d N_c$ by N matrix formed by stacking vertically N_c matrices $\mathbf{A}_i = \mathbf{S}_i\mathbf{F}$ of size N_d by N that include different sensitivity matrices $\mathbf{S}_i$ and the same Fourier operator matrix $\mathbf{F}$ for each coil.

CS reconstruction is based on image compressibility. MR images are compressible, *e.g.*, through a wavelet transform $\mathbf{W}$. To recover the compressed image, the bases of matrices $\mathbf{A}$ and $\mathbf{W}$ are required to be incoherent with each other [1,2]. The incoherency is nearly optimal with a random $\mathbf{A}$. With these requirements fulfilled, the CS reconstruction is calculated by minimizing $\|y - \mathbf{A}x\|_2^2 + \lambda\|\mathbf{W}x\|_1$, where the regularization parameter λ is adjusted according to the noise level $\|e\|_2$. The optimization problem is solved with a non-linear conjugate gradient method based on [3].

The MRI instrument places restrictions on the random structure of $\mathbf{A}$, which is mainly determined by $\mathbf{F}$ and thus by the k-space sampling. We consider two different random k-space sampling patterns. In Figure 1a the k-space is 2D under-sampled by choosing the samples randomly from a rectangular grid as in [3] and [5]. Figure 1b shows a true 2D random under-sampling where the samples are not restricted to a regular grid. Both patterns can be implemented in practice by using a 3D pulse sequence and selecting the sample locations in each plane perpendicular to frequency encoding with the two phase encoding gradients. Furthermore, in order to improve SNR, the k-space sampling should concentrate on the low frequencies that contain the most of the k-space signal energy. The variable density random sampling was implemented in both cases by drawing the k-space radial distance and the k-space angle from the exponential and uniform distributions, respectively.

Fully-sampled brain MRI data with an 8-channel array were used to reconstruct a 128x128 model image x_0 and coil profiles $\mathbf{S}_i$. Using the described two sampling patterns, pMRI measurements with R between 2 and 12 were simulated and noise was added to yield an SNR of $\|\mathbf{A}x_0\|_2/\|e\|_2 \sim 20$ depending on R. CS reconstruction algorithm converged within 15 iterations with Daubechies-4 wavelet as the transform $\mathbf{W}$. The relative error of each reconstructed image x was measured over 20 averages as $\|x_0 - x\|_2/\|x_0\|_2$.

Results

Figure 2 shows the mean relative error and standard deviation of the reconstructions as a function of the acceleration factor R. We see that if the error is kept fixed, true random sampling allows the use of higher R than random sampling from a rectangular grid. To demonstrate the power of combined CS and pMRI, Figure 3 shows the model image (a) and a reconstructed image (b) with a relative error of 0.095, corresponding to R of 8 and 10 for the rectangular and true random sampling, respectively.

Discussion

A truly random sampling pattern improves the CS reconstruction in pMRI. A realization of the method requires only minor changes in the conventional 3D pulse sequences. Further research is needed to optimize the variable density of the sampling pattern. We are in the process of developing pMRI for very low-field MRI, where reductions in measurement time are desperately needed.

References

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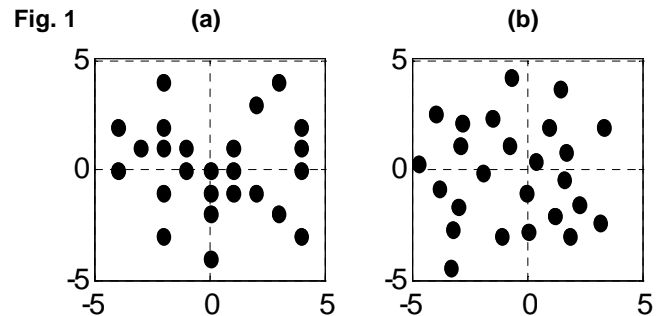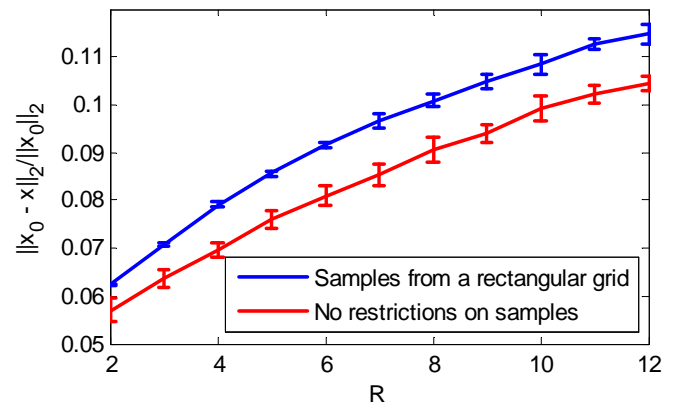
Fig. 1

Fig. 2

Fig. 3