PROSPECTIVE SNR OPTIMIZATION IN *K-T*-BASED SENSITIVITY-ENCODED DYNAMIC IMAGING USING A FAST GEOMETRIC ALGORITHM

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Introduction MR data acquisition and image reconstruction are typically formulated using Fourier transform (FT) theory, i.e., the classical *k*-space relationship. However, in dynamic MR imaging (e.g., 2D Cartesian sampling) it takes a few milliseconds from collecting one k-space phase-encode (PE) line to the next, i.e., sampling in the PE direction is time-sequential and not instantaneous [1]. Hence, a more accurate model for MR data acquisition is the so called "*k-t*-space perspective" (*k*=spatial frequency, *t*=time) [1,2]. Most well known *k-t*-based schemes use *k-t* sampling patterns that are restricted to a lattice (generalized sheared grid) [1-3]. Based on sampling theory, *k-t* sampling on a lattice will result in aliasing of the object's spectrum in the reciprocal domain, referred to as dual *k-t* domain [4-6] or the *x-f*-space (*x*=PE direction, *f*=temporal frequency) [3]. The replication pattern is on the corresponding point-spread function (PSF) which is the FT of the sampling lattice [2,4]. The acceleration in *k-t*-based techniques results from multifold undersampling that the *k-t* lattice provides relative to the Nyquist rate. In UNFOLD [3] and PARADIGM [4], the *k-t* lattice is designed so that there is no overlap of the support region in *x-f*-space. On the contrary, in UNFOLD-based parallel imaging (e.g., TSENSE), k-t SENSE, and PARADISE [5-6], there is overlap in *x-f*-space which can potentially be undone using sensitivity encoding (SE) and prior information (signal model). The prior information is in form of statistics for k-t SENSE, cross-shaped *x-f* support model (Fig 1a) for UNFOLD [3], and multi-banded patient-adaptive *x-f* support model (Fig 1b) for PARADISE [4-6]. Optimizing the reconstruction SNR in this context has been previously introduced and its significance has been demonstrated *in-silico* for PARADISE [5]. A similar study has been conducted for k-t SENSE [7]. We focus on *nongated* cardiac MR techniques that use *x-f* support models and SE, namely, PARADISE [5-6] or UNFOLD-based parallel imaging. We propose a novel algorithm for designing the SNR-optimal *k-t* sampling pattern and study its performance *in-vivo*. The proposed algorithm is geometric, i.e., it is based on geometry of overlap patterns in *x-f* space and does not require any knowledge of coil sensitivity profiles.

Theory Fig 1a depicts a *x-f* support model *B* similar to the one used in UNFOLD [3] characterized by a dynamic FOV (DFOV) and a temporal bandwidth that is varying in *x*. Fig 1b shows a more refined *x-f* support used in PARADISE [6], referred to as the banded support model [4], which formulates the approximate heart motion periodicity (hence the gap between bands) and also models heart-rate (HR) variability [6]. Given a fixed spatial resolution, a *k-t* lattice Λ shown in Fig 2a is parameterized by (*TR*, Δ*k*) and can be expressed as sum of delta functions located on a sheared grid [2,5]. The PSF for Λ, denoted by Λ^* is given by its 2D FT (inverse FT in *k* and forward in *t*). Fig 2b depicts two representative replicas of the *x-f* support (Fig 1a), one centered at the origin and another on a grid point Λ^*(m,n) of the PSF lattice Λ^*. As a result of this replication, pixels within *B* overlap with each other and hence need to be un-mixed. The reconstruction task is to recover all pixels within support *B* by un-mixing the overlapped pixels in *x-f*-space. This overlap pattern is depicted in Figs 3a and 3b for two PSF lattices. In parallel *k-t*-based techniques that only use *x-f* support information (unlike k-t SENSE), this procedure relies on the SE along *x* (no encoding along *f*) provided by multiple receiver channels. If two overlapping pixels have close *x*-coordinates, as shown in Fig 3a, then they are experiencing almost similar SE. Hence, the underlying SENSE equations in *x-f*-space would be closely dependent which will result in an ill-conditioned matrix equation for un-mixing of the overlapped pixels. Therefore, compared to Fig 3a, the overlap pattern in Fig 3b is expected to have a better conditioning (lower spatio-temporal g-factor [5,7]) and result in a better SNR. Motivated by this observation, for a pixel (x_0,f_0) in *B*, we denote by $p(x_0,f_0)$ the minimum *x*-coordinate distance between the set of pixels that overlap onto (x_0,f_0). Let $\rho(TR,\Delta k,B)$ denote the minimum of $p(x,f)$ among all pixels within *B* resulting from sampling on Λ parameterized by (*TR*, Δ*k*) (Fig 2a). We postulate that by designing Λ so that $\rho(TR,\Delta k,B)$ is maximized, the resulting overlap patterns will correspond to well-conditioned (low g-factor) SENSE matrix equations for *x-f*-space reconstruction. Eq. (1) provides a functional form for the optimization problem. Fig 4 summarizes the computational steps for evaluating the cost function for a set of feasible (*TR*, Δ*k*) choices.

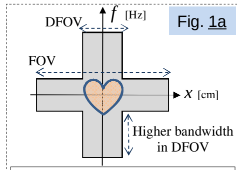

Fig. 1a UNFOLD *x-f* Support (notation: *B*)

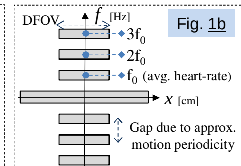

Fig. 1b PARADISE Banded *x-f* Support

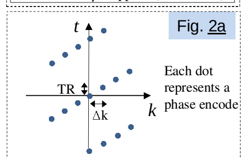

Fig. 2a *k-t* Lattice Λ; Only one PE is acquired during each TR (Λ is time-sequential)

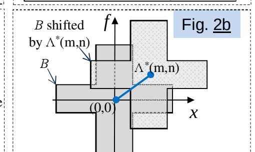

Fig. 2b Replication of *x-f* support *B*; Λ^* is the PSF Lattice

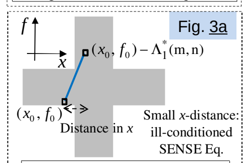

Fig. 3a Bad SNR case: Two overlapping pixels in support *B* with PSF Λ_1^*

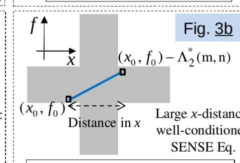

Fig. 3b Good SNR case: Two overlapping pixels in support *B* with PSF Λ_2^*

Methods MR imaging with informed consent was performed under the NHLBI IRB using a 1.5T Siemens Avanto scanner with a 32-element cardiac array. Initially, a gated segmented SSFP cine scan was acquired (|FOV|=420mm square, matrix=256x224, 30 phases, TR=3.5ms, temporal resolution=28ms, GRAPPA rate 4). A customized SSFP pulse sequence was developed to allow for operator-defined TR and ordering of phase-encodes -- hence capable of acquiring a general *k-t* lattice (Fig 2a). MR data for the non-gated PARADISE scheme [5] with a 320x256 image matrix (1.3x1.6mm resolution) was collected during a single breathhold (scan time=16.4s). The *x-f* support model was chosen to be the banded model [5] as in Fig 1b with 11 bands (DC centered) and the following parameters: (1) DFOV location was estimated from localizer scans (|DFOV|=0.32*|FOV|) (2) Average HR during the gated scan was used as an estimate for f_0 (=1.1Hz) (3) The thickness of the bands (Fig 1b) was set to 0.33*f_0 to account for HR mis-estimation and variability during the scan. The proposed *k-t* lattice design algorithm (Fig 4) was run to find the *SNR-optimal lattice* by maximizing ρ (result: ρ^*=0.24*|FOV|). For comparison, a *suboptimal lattice* was computed so that its ρ would satisfy $\rho \approx \rho^*/3$ (result: ρ=0.09*|FOV|). The search space for (*TR*,Δ*k*) was limited according to the acquisition and SSFP pulse sequence specifications (resolution, min TR, etc). Computation time was less than 30s and each *k-t* lattice was immediately preceded by a conventional coil-profile calibration acquisition (128 Nyquist-spaced phase-encodes) to provide the final *k-t* sampling schedule.

$$(\mathrm{TR}^*, \Delta k^*) = \underset{(\mathrm{TR},\,\Delta k)\ \text{feasible}}{\arg\max}\ \rho(\mathrm{TR}, \Delta k, B) \qquad \text{Eq. (1)}$$

Fig. 4 Flowchart for computing the cost function in Eq. (1)

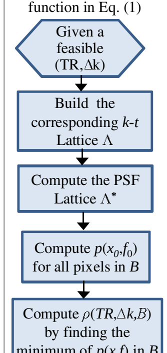

Fig. 5 Gated Cine ED Frame; 256x224 matrix;

Fig. 6a

Fig. 6b

Fig 6 PARADISE Reconstruction; Non-gated; 320x256 matrix; (a) Suboptimal; (b) Optimized

Discussion & Conclusion Fig 5 shows the end-diastolic (ED) frame for the gated cine. Reconstructed ED frames for PARADISE acquired (non-gated) according to the SNR-optimized and suboptimal *k-t* lattices are shown in Figs 6a and 6b, respectively. The optimized lattice acquisition results in a much better reconstruction SNR (Fig 6b) because its underlying SENSE equations (in *x-f* space [5]) are better conditioned (have lower g-factor). In conclusion, the SNR difference seen in Fig 6 demonstrates the effectiveness and significance of the proposed *k-t* lattice design technique. In contrast to previous *k-t* acquisition design techniques [5-7], all computations in the proposed algorithm are geometric-based and independent of coil sensitivities. This feature enables fast computation of the SNR-optimal *k-t* sampling schedule for *k-t*-based parallel imaging prior to running the MR scan.

References [1] Liang, Lauterbur, IEEE Med. Imaging 13:677-86, 1994 [2] Willis, Bresler, IEEE Info. Theory 43:190-207, 1997 [3] Tsao, MRM 47:202-07, 2002 [4] Aggarwal *et al*, Inverse Problems (24) 2008 [5] Sharif, Bresler, Proc IEEE ISBI'07, 1020-23 [6] Sharif *et al,* ISMRM'07(15), p151 [7] Malik *et al*, ISMRM'08(16), p11

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