# Prewhitening and Maximum Likelihood Estimation for Root-Sum-of-Squares Images

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**INTRODUCTION:** In recent years, phased array coils have seen widespread adoption across a variety of MR applications. Multiple receivers can be exploited to improve SNR and/or reduce scan time through parallel imaging methods. Although the matched-filter is the theoretically optimal method for combining coils (highest SNR and no bias) [1], it requires accurate coil sensitivity maps and access to complex image data. Since these are not always available, the alternative Root-Sum-of-Squares (RSS) is the most commonly used method to combine signals originating from different coils.

Similar to single-channel or quadrature reception, the assumption of Gaussian noise is valid for RSS images at high SNR. At low SNR, there is a magnitude bias that increases with the number of coils [2]. This can cause systemic errors in quantitative measurements that depend on data points close to the noise floor (e.g. Diffusion, Relaxometry, Magnetization Transfer, etc.).

**THEORY:** When the noise covariance matrix, $\mathbf{R_n}$, is a multiple of the identity matrix, RSS images follow a noncentral $\chi^2$ distribution [2]. The likelihood function represents the joint probability for a single voxel imaged at multiple time points or for several voxels within a uniform ROI:

$$(1)\quad L(A) = \prod_{i=1}^{N} \frac{A}{\sigma^2}\left(\frac{M_i}{A}\right)^n e^{-[(A^2+M_i^2)/2\sigma^2]} I_{n-1}\left(\frac{M_i A}{\sigma^2}\right)$$

where $A$ is the signal amplitude (which we are trying to estimate), $\mathbf{M}$ is a vector of RSS magnitudes, $n$ is the number of channels, and $\sigma$ is the average noise in each channel. In situations where the coils are highly correlated or have unequal noise power, Eq. 1 is not valid for the standard RSS combination. Instead we can use the prewhitened RSS [1] for which Eq. 1 still applies:

$$(2)\quad \mathbf{RSS_{prewhitened}} = \sqrt{\mathbf{p}^{\mathbf{T}}\mathbf{R_n^{-1}}\mathbf{p}^{*}}$$

Maximum Likelihood (ML) estimation of signal amplitude from single-channel magnitude data has been reported previously [3], although to our knowledge, it has not been extended to the multichannel case. We seek to estimate the signal amplitude that maximizes L(A). Estimation of the noise parameter is unnecessary, since $\sigma$=1 for prewhitened images.

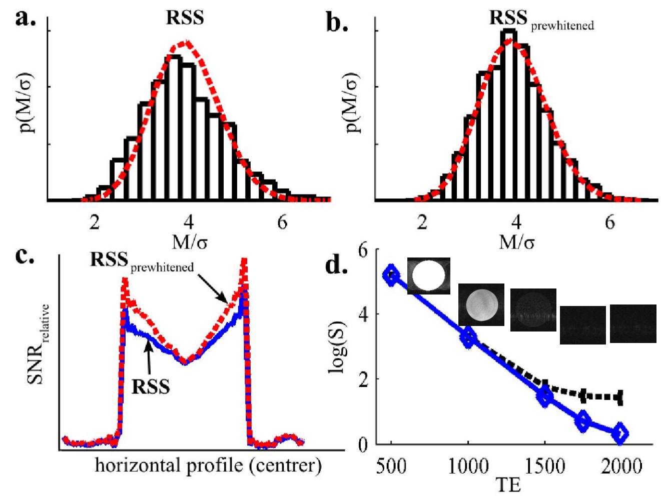


**Figure 1.** Background noise statistics for (a) standard and (b) prewhitened RSS images. The noncentral $\chi^2$ distribution is plotted as a dashed line. (c) Relative SNR for the center profile. (d) $T_2$ decay curve plotted on a log scale. The dashed line represents the mean signal in the ROI (1 s.d. error bars), while the solid line shows the ML fit.

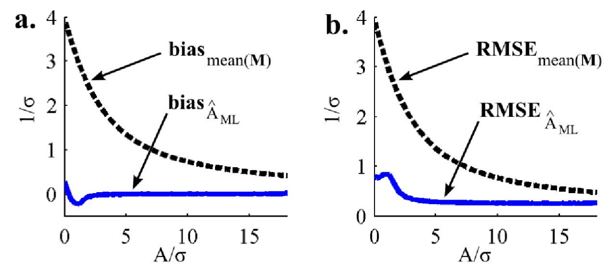


**Figure 2.** Simulated comparison of (a) bias and (b) root-mean-squared error (RMSE) for the ML estimate versus the mean signal magnitude over a range of SNR values (for a 16 voxel sample size).

**METHODS:** $\mathbf{R_n}$ was measured from a reference scan as in Ref. 4. Four spin-echo images (TE=500, 1000, 1500 and 1750 ms, TR=2s) of a spherical QA phantom were acquired on a 3T GE Signa scanner with an 8-channel GE head coil. $A_{ML}$ was estimated at each TE from a 4x4 voxel region of interest at the centre of the phantom. ML estimation was performed using the Nelder-Mead simplex algorithm in MATLAB (Mathworks, Natick, MA). Monte Carlo simulations were used to evaluate the performance of the Maximum Likelihood estimator over a range of SNR values.

**RESULTS & DISCUSSION:** Figure 1a and b demonstrate that the background signal statistics from prewhitened RSS are in agreement with the noncentral $\chi^2$ distribution, while the standard RSS images show a slight but noticeable deviation. The prewhitened images also exhibit an SNR improvement of up to 22% relative to the RSS images (Figure 1c). The log-scaled $T_2$-decay curve in Figure 1d provides an example of the ML estimator's ability to recover signals from below the noise floor. Simulation results in Figure 2 compare the bias and root-mean-squared for the ML estimator versus the mean magnitude. ML fitting may also be performed without prewhitening in situations where the individual coil images are unavailable (provided that the coils are sufficiently decoupled).

**CONCLUSION:** If reliable coil sensitivity information is available, matched-filter based techniques should be the preferred choice for recovering signals from low SNR regions because of their superior precision [1,5]. However, the combination of prewhitening and ML estimation is a pragmatic alternative to reduce magnitude bias in RSS images.

**REFERENCES:**[1] Roemer (1990). *MRM* **16**(2),192-225 [2] Constantinides (1997). *MRM* **38**(5), 852-857 [3] Sijbers (2004). *MRM* **51**(3), 586-594 [4] Kellman (2005) *MRM* **54**(6), 1439-1447 [5] Bydder (2002), *MRM* **47**(3), 539-548