

# Creation of Arbitrary Spatial Harmonics through the Combination of Orthogonal Weights (CASHCOW): A Generalized Direct GRAPPA Approach for Non-Cartesian Data

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**Introduction:** Non-Cartesian parallel imaging is complicated by the varying locations of acquired points with respect to each Cartesian k-space point. Many different reconstruction methods have been proposed; however, these are either iterative [1], require a specific trajectory symmetry [2], or are time consuming due to the large number of different weights that must be calculated [3]. It has been shown that GRAPPA Operators [4] for vertical and horizontal shifts can be combined to perform shifts in arbitrary directions over small distances (GROG [5]), although this method does not provide accurate reconstructions over large shift distances because only one source point per target point is used. The method proposed here uses the GRAPPA Operator formalism to generate multi-point weight sets, allowing the reconstruction of Cartesian points from the arbitrarily located undersampled non-Cartesian datapoints. The problem is solved by formulating its reverse: weight sets are generated which would perform a reconstruction of the non-Cartesian points using a given Cartesian target point pattern. Once this weight set has been found, its inverse can be used to reconstruct the desired Cartesian point from the non-Cartesian points. This method is called Creation of Arbitrary Spatial Harmonics through the Combination of Orthogonal Weights (CASHCOW), and it provides a link between GRAPPA (with the possibility of large shift distances) and GROG (with flexible weight set combinations). The reconstruction of undersampled radial data using CASHCOW is demonstrated here.

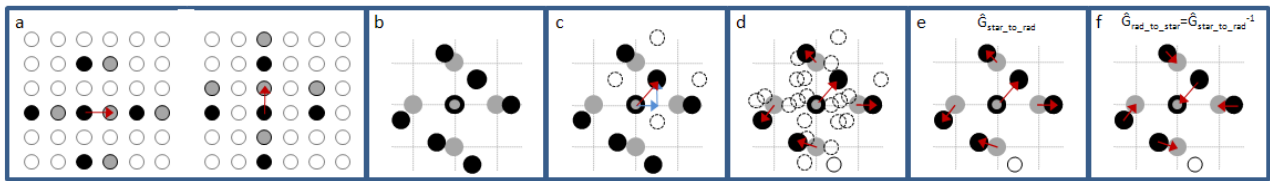


Figure 1: A schematic of the CASHCOW calibration and application for a star-shaped weight set, as described in the text.

**Theory:** The creation of weights for the reconstruction of Cartesian points using non-Cartesian points involves the following steps (see Fig. 1):

- Weights for both the horizontal and vertical shifts are calculated using a given pattern
- Cartesian target points (gray) and non-Cartesian source points (black) are identified in the data; the central source point in black and gray is the primary Cartesian point to be reconstructed
- The weights needed to shift the Cartesian pattern to one of the non-Cartesian target points (red shift arrow) are calculated by combining horizontal and vertical weights (calculated in step a, blue arrows) using the power of the appropriate distances
- Step c is repeated for each Cartesian point in the pattern. The weights for many additional irrelevant points are also generated
- Only those weights which actually correspond to the non-Cartesian points are retained (forming  $G_{\text{star\_to\_rad}}$ )
- The inverse of this weight set is taken (forming  $G_{\text{rad\_to\_star}}$ ), allowing the selected Cartesian point (in gray and black) to be reconstructed from the acquired non-Cartesian datapoints

**Methods:** In vivo head data were acquired using a radial SSFP sequence with the following dimensions: Matrix size: 256x256, Projections: 256, Read points: 512, Channels: 12. The datasets were retrospectively undersampled to yield R=2, R=3, and R=4 datasets, with 128, 88, and 64 projections, respectively. The central Nyquist sampled portion of each dataset was gridded, and this Cartesian k-space was used to calibrate the weight sets. For all of the images shown, two patterns were used; the star shown in Fig 1 and a similar “X” pattern.

**Results:** The results of the CASHCOW reconstructions for the R=1 to R=4 datasets are shown in Figure 2. Despite the radial undersampling, the only difference in these images is the noise level, which increases with larger acceleration factors.

**Discussion and Conclusion:** CASHCOW is a novel method which uses the GRAPPA Operator concept to reconstruct images from undersampled non-Cartesian datasets. The needed weights for arbitrary k-space point patterns are found by first generating weights which move Cartesian points to a given non-Cartesian pattern, and then inverting this weight set to move from the acquired non-Cartesian points to the missing Cartesian k-space locations. Using only two such patterns, artifact-free images could be created using radial datasets with undersampling factors up to R=4. CASHCOW is advantageous because it is the first GRAPPA method which allows large shifts between points of arbitrary locations in k-space; in all other GRAPPA methods, the source point pattern is fixed, making its application to non-Cartesian parallel imaging challenging. CASHCOW can be seen as the connection between GRAPPA and GROG in that multiple source points are used to fit single target points leading to more stable reconstruction results with larger shift distances (as in GRAPPA), and that shifts of different distances and directions can be calculated from a single weight set (GROG). CASHCOW can be used for robust data gridding or as a simple, one-step alternative to other non-Cartesian GRAPPA methods.

- References:**
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  - [5] Seiberlich N, et al. MRM 2007 Dec;58(6):1257-65.

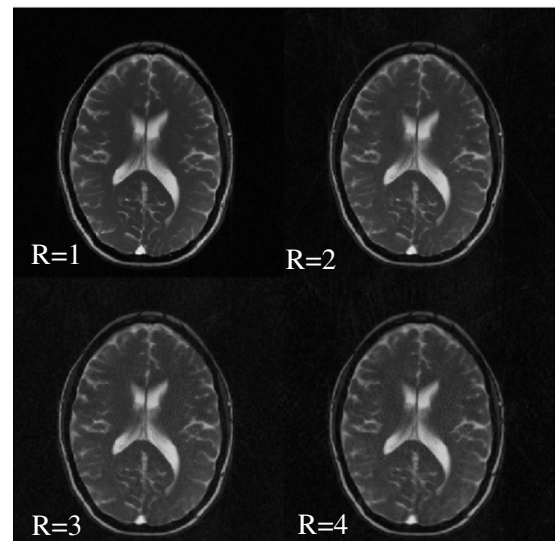


Figure 2: In vivo radial dataset reconstructed with CASHCOW for acceleration factors from R=1 to R=4.

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