Quantitative g-factors for radial GRAPPA

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Introduction:

Recently, a theoretical framework for quantitative estimation of the non-uniform noise enhancement in GRAPPA reconstructions has been derived [1,2]. These works show that a GRAPPA g-factor can be calculated directly from the GRAPPA weights, which corresponds with the g-factor familiar in SENSE reconstructions [3]. In this work, we extend this concept to GRAPPA acquisitions where multiple GRAPPA kernels are involved in the reconstruction process, which is generally the case in non-Cartesian GRAPPA. The feasibility of this approach is demonstrated for R=4 radial GRAPPA.

Theory:

In previous work, we derived a quantitative expression for calculating the noise enhancement (g-factor) in Cartesian GRAPPA reconstructions directly from the GRAPPA reconstruction weights according to Eq 1 on a pixel by pixel basis (1).

$$(1)\quad g = \frac{SNR^{full}}{SNR^{acc}\cdot\sqrt{R}} = \frac{\sqrt{\left|(a\cdot\mathbf{W})\cdot\boldsymbol{\Sigma}^2\cdot(a\cdot\mathbf{W})^H\right|}}{\sqrt{\left|(a\cdot\mathbf{1})\cdot\boldsymbol{\Sigma}^2\cdot(a\cdot\mathbf{1})^H\right|}}$$

In this equation the weight matrix $\mathbf{W}$ contains the GRAPPA weights in image space which can be derived from the GRAPPA reconstruction kernel in k-space [4,5] and $\boldsymbol{\Sigma}$ represents the noise correlation matrix accounting for potential correlations between the individual receiver channels. The coil combining coefficients in the vector a, needed for g-factors in the case of combined GRAPPA images, can simply be determined either from the low resolution ACS data or the high resolution reconstructed GRAPPA images.

In the case of reconstructions where multiple kernels are needed, one can show that the final g-factor can be expressed as:

$$(2)\quad g = \frac{\sqrt{\sum_{k=1}^{N} f_k\cdot R_k\cdot g_k^2}}{\sqrt{R}} \quad and \quad R = \left(\sum_{k=1}^{N}\frac{f_k}{R_k}\right)^{-1}$$

where k runs from 1 to number of GRAPPA kernels N used in the reconstruction, f_k represents the fraction of k-space in which the individual GRAPPA kernel was applied, R_k is the corresponding reduction factor of the kernel, and R the overall reduction factor. In radial GRAPPA, the k-space is segmented in the read-out and angular directions (see Fig 1). Within each segment the data is treated as Cartesian and standard Cartesian GRAPPA is performed to reconstruct the missing data. While the reduction factors and the kernel geometries are the same for each segment, the kernel orientation changes along the angular direction and the effective undersampling factor changes along the read-out direction. Thus, prior to the combination of the individual g-factors derived using Eq. 1, the g-factors must be rotated according to the angular orientation and stretched according to the segment's position along the read-out.

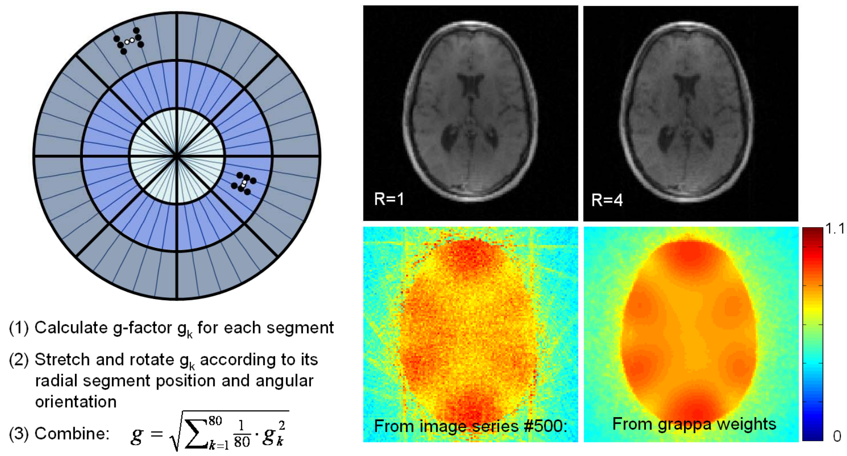

Fig.1: Schematic of the segmentation procedure in Radial GRAPPA. In each segment a different GRAPPA kernel is required (top left). Image reconstructions of fully sampled (R=1) radial data and (R=4) accelerated data after Radial GRAPPA (top right). In addition, quantitative g-factor maps for R=4 Radial GRAPPA are given, derived from an image series and directly from the GRAPPA weights (bottom right).

Materials and Methods:

In order to validate this concept, a fully encoded radial acquisition was simulated with 200 radial projections and 184 readout points using Cartesian data acquired with an eight-channel receiver coil. Using this dataset, a 4-times undersampled dataset was mimicked by retrospectively removing projections and reconstructed using radial GRAPPA with 80 segments (10 in angular direction and 8 in radial direction) and a 2x3 GRAPPA kernel. The resulting radial k-space was gridded using filtered back projection. The g-factor for the radial GRAPPA reconstruction was calculated directly from the corresponding GRAPPA weights as described in the theory section. In addition, reference g-factor maps were derived using the pseudo multiple replica approach [6,7]. This approach requires a fully encoded radial data set and a noise only image in order to mimic 500 unaccelerated and accelerated radial acquisitions. On each artificially created dataset radial GRAPPA reconstruction is performed and the SNR is calculated for the accelerated and unaccelerated case by taking the mean and the standard deviation of the image series.

Results:

In the Figure, the fully encoded (R=1) reference and accelerated (R=4) radial GRAPPA reconstruction is shown. In addition, the GRAPPA g-factor derived directly from the reconstruction weights for the R=4 accelerated radial GRAPPA reconstruction is displayed and the reference g-factor maps calculated from the pseudo image series. The g-factor map derived directly from the GRAPPA weights are in excellent agreement to the reference g-factor, which required a fully encoded radial data set and a noise only image in order to mimick 500 unaccelerated and accelerated radial acquisitions and GRAPPA reconstructions.

Conclusion:

The GRAPPA g-factor estimation presented here allows one to quickly and accurately estimate the non-uniform noise enhancement in cases where multiple GRAPPA kernels are used. This includes, for example, Cartesian GRAPPA applied to accelerated VD acquisitions with varying reduction factor along the phase encoding direction. In addition, this concept allows one to easily derive g-factors for GRAPPA reconstructions with included ACS data. In general, Eq 2 allows one to calculate quantitative g-factors for GRAPPA reconstruction of arbitrary non Cartesian trajectories directly from the GRAPPA weights. Quantitative GRAPPA g-factors were successfully derived for accelerated PROPELLER, radial, spiral and zig-zag GRAPPA. However, in this abstract we focussed on demonstrating the applicability of this concept to radial GRAPPA reconstructions.

References:

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