SEPARATION OF RELAXATION TIME CONSTANTS THROUGH CYLINDRICAL COORDINATES

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Introduction: The Bloch equations are fundamental to the classical description of the magnetic resonance dynamics which is the basis of MRI [1]. Here, we provide a novel interpretation of magnetic resonance by expressing the Bloch equations in cylindrical coordinates. Interpretation of magnetic resonance in cylindrical coordinates provides a significant simplification of the occurrence of relaxation parameters, T_1, T_2, and T_2^*. Moreover, the expression of the Bloch equations in cylindrical coordinates enables a straight forward description of the Bloch equations in the rotating frame. The novel framework introduced here is directly applicable to research into optimal excitation patterns in magnetic resonance problems [2, 3].

Methods: The Bloch equations in Cartesian coordinates in the laboratory frame of reference are written as

$$\dot{M}_x = -\frac{M_x}{T_2} + \omega_z M_y - \omega_y M_z; \qquad \dot{M}_y = -\omega_z M_x - \frac{M_y}{T_2} + \omega_x M_z; \qquad \dot{M}_z = \omega_y M_x - \omega_x M_y - \frac{M_z}{T_1} + \frac{M_0}{T_1}. \tag{1}$$

If the applied external field is $\mathbf{B}_{ext} = B_1^e(t)\cos(\omega_0 t)\,\hat{\mathbf{i}} - B_1^e(t)\sin(\omega_0 t)\,\hat{\mathbf{j}} + (B_0 + \Delta B_0)\,\hat{\mathbf{k}}$, then by transferring the Bloch equations into the cylindrical coordinates we have

$$\dot{M}_r = -\frac{M_r}{T_2} + \omega_1(t) M_z \sin(\omega_0 t + M_\phi); \qquad \dot{M}_\phi = -(\omega_0 + \Delta\omega) + \frac{M_z}{M_r}\omega_1(t)\cos(\omega_0 t + M_\phi); \qquad \dot{M}_z = -\omega_1(t) M_r \sin(\omega_0 t + M_\phi) - \frac{M_z}{T_1} + \frac{M_0}{T_1}, \tag{2}$$

where $\omega_1(t) = \gamma B_1^{\,e}(t)$ and ω_0 represents the main magnet Larmor frequency, $\omega_0 = \gamma B_0$. In order to transfer the Bloch equations to the rotating frame of reference in cylindrical coordinates it is sufficient to define a new azimuthal component $M_{\phi'} = \omega_0 t + M_\phi$. Subsequently the cylindrical Bloch equations in the rotating frame of reference may be written as

$$\dot{M}_{r'} = -\frac{1}{T_2} M_{r'} + \omega_1(t)\; M_{z'}\sin(M_{\phi'}); \qquad \dot{M}_{\phi'} = -\Delta\omega + \frac{M_{z'}}{M_{r'}}\omega_1(t)\cos(M_{\phi'}); \qquad \dot{M}_{z'} = -\omega_1(t) M_{r'}\sin(M_{\phi'}) - \frac{M_{z'}}{T_1} + \frac{M_0}{T_1}. \tag{3}$$

The prime notation distinguishes the MR signal components in rotating frame from the laboratory frame of reference. When the excitation is turned off Equation (3) simplifies to

$$\dot{M}_{r'} = -\frac{1}{T_2} M_{r'}; \qquad \dot{M}_{\phi'} = -\Delta\omega; \qquad \dot{M}_{z'} = -\frac{M_{z'}}{T_1} + \frac{M_0}{T_1}. \tag{4}$$

Equation (4) clearly separates the relaxation time constants. It is obvious that T_1 relaxation time constant affects the vertical component, while the magnitude of the radial component only depends on T_2. It is important to note that field inhomogeneities only change the azimuthal component of the MR signal. This is the cause of T'_2 decay of the signal. If the field inhomogeneities are assumed to have a Lorentzian distribution, it follows that the MR signal decays with T_2^* time constant, for which $1/T_2^{\,*} = 1/T_2 + 1/T_2^{\,'}$. Thus, from the dynamics of the MR signal in the cylindrical coordinates, the two components of the T_2^* relaxation and their respective sources become immediately apparent. Moreover, it is evident that a π refocusing pulse can compensate the effect of T'_2 but not the intrinsic T_2.

Simulation Results: The simulation result of the cylindrical Bloch equations for a single off resonance isochromat is depicted in Figure 1. The simulation result for the azimuth component of five isochromats is represented in Figure 2 demonstrating rephrasing of the isochromats.

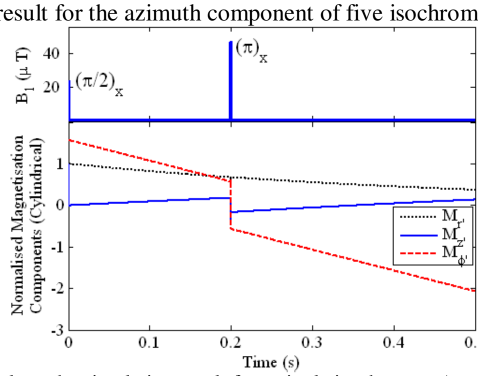

Figure 1. Pulse echo simulation result for a single isochromat, Δω=5rad/s. A π/2 pulse is applied at t=0, and a π pulse is applied at t=0.2s. Clearly, the refocusing pulse does not change the radial component but changes the azimuthal and vertical components. In the simulated spin system T_1=1s, and T_2 = 0.5s.

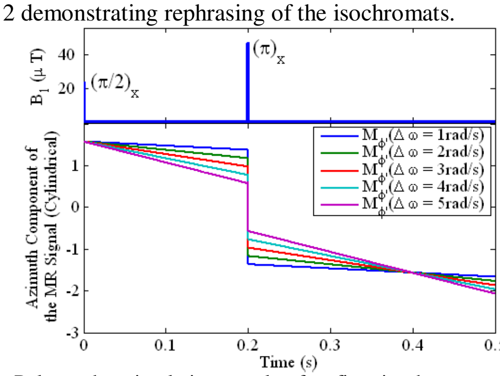

Figure 2. Pulse echo simulation results for five isochromats. Only the azimuthal components are shown in this figure. After applying the refocusing pulse at t=0.2s all isochromats will have the same azimuth value at t= 0.4s. The spin system is the same as spin system in Figure 1.

Conclusions: We have taken a fresh a look at magnetic resonance by transferring the Bloch equations into cylindrical coordinates. In this new framework, the differences between the relaxation processes are immediately evident. It is possible to transfer the cylindrical Bloch equations to an excitation dependent rotating frame of reference [4] and find an approximate analytic solution to the Bloch equations. We expect that the new representation of the Bloch equation will allow researchers to revisit the pulse design question [5] and aid in the determination of optimal excitation patterns [3].

References: [1] Z. Liang and P.C. Lauterbur, *Principles of Magnetic Resonance Imaging, A Signal Processing Perspective,* IEEE Press, 2000. [2] N. Khaneja *et al.*, JMR, 2003, 162:311-319. [3] B. Tahayori *et al.*, *IFAC Cong.,* 2008, 10301-10306. [4] B. Tahayori *et al.*, *IEEE EMBC,* 2008, 5769-5773. [5] J.S. Li and N. Khaneja, *Phys. Rev. A,* 2006, 73:030302.