Two dimensional Spatial Selective Shinnar Le Roux pulse design for arbitrary *k*-space trajectory

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Introduction: The Shinnar Le Roux (SLR) method [1,2,3] has been widely used to design slice selection pulse and two-dimensional (2D) RF pulse on EPI trajectory [4] due to the advantage of allowing tradeoffs among the in-slice ripple, out-of-slice ripple and transition. In this work, the SLR method is extended to design 2D pulse on arbitrary *k*-space trajectory. Firstly, 2D filter coefficients are designed using McClellan transformation; then, the inverse gridding algorithm is applied to resample the 2D filter coefficients in Cartesian trajectory to arbitrary *k*-space trajectory needed. Finally, the SLR inverse transform is applied to convert these filter coefficients to a 2D RF pulse. A 90^o excitation pulse is designed and the excitation profile is simulated to verify our proposed method.

Theory and method: As in the traditional SLR method, the 2D RF pulse is assumed to be piecewise pulse, which is commonly used in commercial MR scanner. Thus, the rotation caused by each RF sample and the corresponding gradients can be treated as spinors, and the state of spin after each rotation can be expressed by a 2×1 complex vector:

$$\begin{pmatrix} A_j \\ B_j \end{pmatrix} = \begin{pmatrix} C_j & S_j^* z^{-1} \\ S_j & C_j z^{-1} \end{pmatrix} \begin{pmatrix} A_{j-1} \\ B_{j-1} \end{pmatrix} \quad (1),$$

where C_j and S_j denote the rotation caused by the *j*th RF sample, A_j and B_j can be regarded as the spin state after *j*th rotation [3]. z^{-1} denotes the rotation caused by gradients:

$$z = e^{i\gamma(G_x x + G_y y)\Delta t} \quad (2),$$

where G_x and G_y denote the *x*- and *y*- gradients respectively. Given the equilibrium state $\begin{bmatrix} A_0 \\ B_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, the spin state after *N*-length RF pulse can be calculated by recursion:

$$A_N(z_j) = \sum_{j=0}^{N-1} a_j \prod_{s=1}^{j} z_s^{-1} \quad (3) \quad \text{and} \quad B_N(z_j) = \sum_{j=0}^{N-1} b_j \prod_{s=1}^{j} z_s^{-1} \quad (4).$$

Here, if we assume that the gradients are constant, the trajectory is regarded as Cartesian. Therefore the Eq.(3) and (4) can be expressed as two polynomials as following:

$$A_{carN}(z) = \sum_{j=0}^{N-1} a_{carj} z^{-j} \quad (5) \quad \text{and} \quad B_{carN}(z) = \sum_{j=0}^{N-1} b_{carj} z^{-j} \quad (6).$$

The coefficient b_{carj} and a_{carj} are hence expressed as $b_{m\times m}$ and $a_{m\times m}$ (where $m\times m = N$) which denote they are along Cartesian trajectory. The $b_{m\times m}$ can be designed using 2D Finite Impulse Response (FIR) filter design algorithm such as McClellan transform. To calculate the coefficients $a_{m\times m}$ it is necessary to use the relationship between $A_{carN}(z)$ and $B_{carN}(z)$:

$$A_{carN}^2(z) + B_{carN}^2(z) = 1 \quad (7).$$

As long as the coefficients $b_{m\times m}$ and $a_{m\times m}$ are obtained, the RF pulse can be designed using inverse SLR transform. The procedure for designing 2D SLR pulse on arbitrary *k*-space trajectory is shown in figure 1. Firstly, we assume that the gradients are constant and specify the excitation profile errors. Secondly, the coefficient $b_{m\times m}$ is designed using Park-McClellan algorithm and McClellan transform. By taking 2D Fourier transform of $b_{m\times m}$ the $B_{carN}(z)$ can be obtained. Using the relationship in Eq.(7) we can get $A_{carN}(z)$ and then obtain $a_{m\times m}$ by taking 2D inverse Fourier transform. Here the coefficients $b_{m\times m}$ and $a_{m\times m}$ are both in Cartesian trajectory, so inverse gridding must be applied to resample these coefficients to b_s and a_s on other trajectory we need. Finally, the RF pulse can be calculated by taking inverse SLR transform of b_s and a_s.

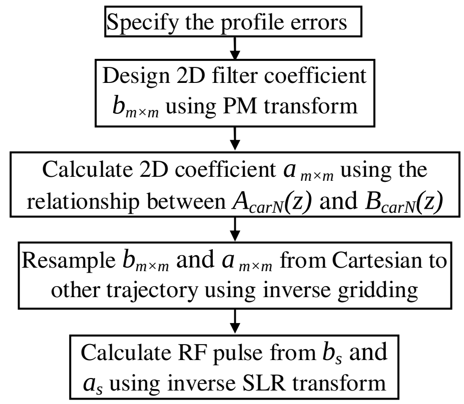

Figure 1 Design procedure for 2D SLR pulse on arbitrary *k*-trajectory.

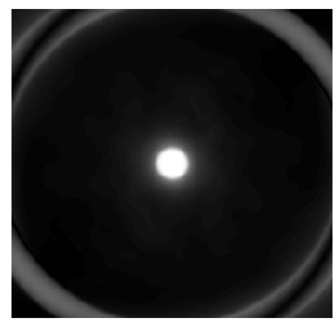

Figure 2 2D excitation profile.

Simulation and results: A 90^o excitation pulse was designed and the profile was simulated using Matlab. The target profile was a 6cm diameter cylinder at the center of a 33cm×33cm FOV. A spiral trajectory with 18 turns and maximum *k*-space value of 0.5cycle/cm was used. The specified in-slice error and out-of-slice error were 0.003 and 0.05 respectively. The 2D and 1D excitation profiles are shown in figure 2 and figure 3. It is illustrate that the first sidelobe is about 16 cm away from the main lobe. The in-slice error is below 0.003 which is no greater than the specified values. The magnified in-slice profile shown in figure 4 illustrates the profile error clearly. It is noticed that the transition is not very sharp. This is due to the error caused by inverse gridding.

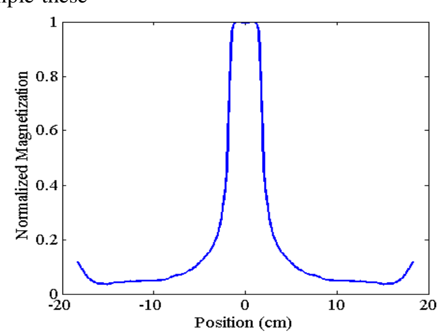

Figure 3 1D excitation profile.

Conclusion and discussion: In this work, the SLR method has been extended to design 2D spatial selective pulse on arbitrary *k*-space trajectory. The filter coefficients are first designed in Cartesian trajectory using the McClellan transformation and then resampled to other trajectory by inverse gridding. These resampled coefficients can be used for designing 2D RF pulse by taking inverse SLR transform. Our proposed method allows tradeoffs between the in-slice ripple, out-of-slice ripple and the transition. However the transition is not very sharp due to the error caused by inverse gridding. This is a preliminary work. The error caused by inverse gridding, as well as the relationship between the gridding parameters and the filter design parameters, will be analyzed.

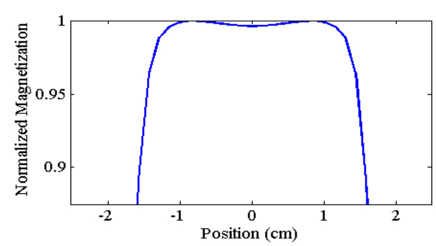

Figure 4 Magnified in-slice profile.

References: [1] Shinnar M, et al, Magn Reson Med 1989; 12: 74-80. [2] Shinnar M, et al, Magn Reson Med 1989; 12: 81-87. [3] Pauly JM, et al, IEEE on Medical Imaging 1991; 10: 53-65. [4] Pauly JM, et al, Magn Reson Med 1993; 29: 776-782.

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