Sparse Selective Excitation Pulse Design Using Adaptive Energy Threshold Method

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Introduction

In conventional pulse design methods, one designs the RF waveform with pre-determined k-trajectory (hence the gradient waveform)[1]. Traditionally one design the k-trajectory based on the k positions required by Nyquist theorem. However Nyquist theorem is a sufficient but not necessary condition for most practical target profile. Unlike the spatial encoding task in imaging, where the underlying image is unknown [2], in the case of excitation pulse design the target excitation profile is known *a prior*i [3,4]. This provides a Bloch equation-based RF pulse design with great opportunities for optimization for the k space sparsity. In this work we propose a method that exploits this prior-knowledge using Adaptive Energy-Threshold.

Theory

While the conventional pulse design with given k positions can be seen as a Least-Squares (LS) problem, pulse design with the k points to be sparsified at the same time leads us to the so-called sparse-approximation problem [5,6,7]. One can approach this type of problems by either solving L1 regularized LS problem [4] or take a greedy-wise approach [5,6,11]. If the target profile has sparse representation in Fourier domain, one can end up with much smaller number of k positions than required by Nyquist theorem.

If one assumes the k positions required by Nyquist theorem as initial k position reservoir, from which one wish to find the subset with fewest members to represent the target profile given an error tolerance (TOL), one can show: because of the orthogonal property of the Nyquist-Fourier-harmonics, the general greedy approach is (i) optimal concerning sparsity, (ii) equivalent to a method of Adaptive Energy Threshold, which has advantages in terms of computational cost compared with general greedy approach.

Algorithm: (Adaptive Energy Threshold)

Notations: P is the target profile in vector form, $\phi_k(x)$ is the Fourier harmonics regarding frequency k, b_k is the projection of P on ϕ_k, m is the number of the chosen k positions, res_m is the residual using the m chosen k positions. G_{max} and S_{max} are the maximum gradient and slew rate constraints.

Step-I: Calculate the projection of the target profile P on all Nyquist Fourier harmonics: $\phi_k(x) = e^{ikx}$.
The projection $|b_k|^2$ has the physical meaning of RF energy at the corresponding k position.

$$|b_k|^2 = \langle P, \phi_k \rangle$$

Step-II: Rank the Nyquist k positions according to descending $|b_k|^2$

Step-III: Calculate the residual with the first m k-positions:

$$res_m = (\sum_i \phi_{ki} * |b_{ki}|^2) - P$$

Step-IV: *if* $|res_m|^2 >$ TOL, $m = m+1 \rightarrow$ III)
else , $\rightarrow$ V)

Step-V: Design the k trajectory traversing the chosen k positions with G_{max}, S_{max} constraints.

Step-VI: Redesign RF with the resulting k trajectory using a conventional LS pulse design.

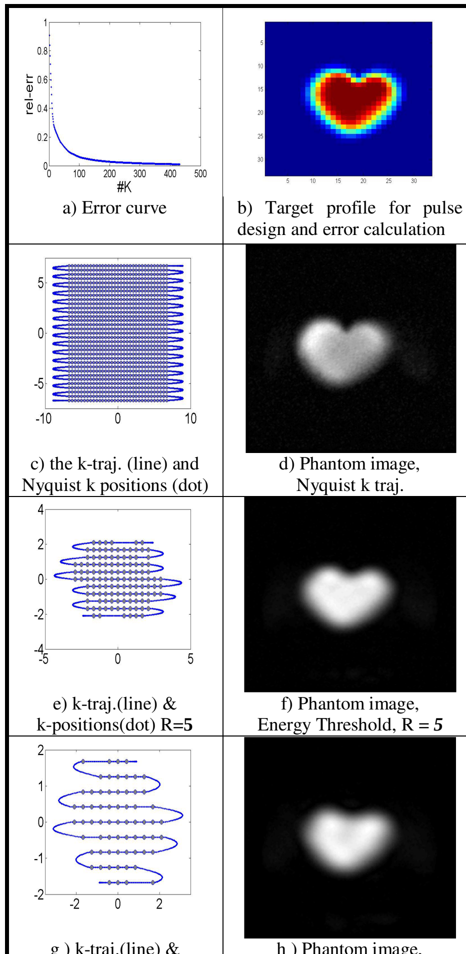
In step-III, Parseval's theorem allows further reduction of computational cost: $|res|^2 = (\sum_{i=N+1:M} |b_{ki}|^2)$.

In step-V, we at first connect the locations in a suboptimal EPI-like manner, and then redesign the k trajectory that traverses the ordered k positions with the pursuit of the shortest time duration, which is a non-trivial task. There are on-going efforts along a related direction [8,9]. In our experiment we use a suboptimal routine based on the method proposed in [8].

Results

We validated the energy threshold method by phantom experiment on a GE 3T scanner using a sphere phantom. The target is a 2D heart shape profile (Fig-b).

Fig-a) shows the error-curve as a function of the number of k positions selected. This curve shows a exponential decay. By choosing different TOL one can make flexible tradeoff between the excitation quality and time optimality. The K/RF joint pulse design was performed in 3 different sparse levels. (Fig-c,d) show the k-trajectory and the excitation image using all k positions required by Nyquist theorem(according to a spatial resolution 33x33). We take that as the reference. (Fig-e,f) show the results obtained with the Energy Threshold method using a design-TOL=5%. The algorithm ended up with 121 k positions (the gray dots in fig-e). The resulting k trajectory (Fig-e, line) was with a reduction factor R = ***5.1*** over the Nyquist one (Fig-c, line). (Fig-g,h) show the results from using design-TOL =10%, the algorithm used 65 k positions. The resulting k trajectory was with a reduction factor of ***7.3***.



a) Error curve

b) Target profile for pulse design and error calculation

c) the k-traj. (line) and Nyquist k positions (dot)

d) Phantom image, Nyquist k traj.

e) k-traj.(line) & k-positions(dot) R=**5**

f) Phantom image, Energy Threshold, R = ***5***

g) k-traj.(line) & k-positions(dot) R=**7.3**.

h) Phantom image, Energy Threshold, R = ***7.3***

Figure

Conclusion & Discussion

Practical selective excitation target profiles do not require full sampling of a Nyquist-k-grid. The present method of sparsifying the necessary k-points using the prior-knowledge of individual excitation target profile may significantly reduce the pulse duration. This method allows also a flexible tradeoff between the excitation quality and the pulse duration. The method was validated in phantom imaging experiments.

In principle there are three fundations for sparsifying the k positions: A) the frequency domain sparsity of the excitation target profile, B) additional degree of freedom created by parallel RF transmit [12,13], and C) relaxing the phase constraints of the target profile [10]. The Energy Threshold method presented here concentrates on exploiting A). Combining A) & B) will give further improvement concerning sparsity. A method aiming at that is presented in a separate work. We expect further significant improvement of the sparsity by possibly integrating the phase relaxing mechanism into our methods.

This method could be used to help determine the optimal number and positioning of the spokes for 3D spokes pulse design [4].

The optimality concerning sparsity of this method was established by assuming Nyquist-k-grid as the initial k-position-reservoir. From our empirical tests, we noticed that the sparsity improvement by going beyond Nyquist-k-grid as the initial reservoir is not significant.

References

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