Fast selection of phase encoding locations in parallel excitation

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Introduction

We propose a novel, fast method based on a hybrid version of Simultaneous Orthogonal Matching Pursuit(S-OMP)[1] to select sparse phase-encoding locations in a Echo-Volumar(EV) trajectory for parallel excitation pulse design with slice-selective subpulses [2]-[3]. In [2]-[3], the pulse length is dominated by the number of phase-encoding locations in the EV trajectory, so sparse phase-encoding is critical to generating a short RF pulse. This problem of enforcing sparsity in phase-encoding locations was presented as a convex optimization problem in a Second Order Cone Program(SOCP) form and solved in [3]. Unfortunately, typical SOCP routines with parallel excitation setting are too slow to be computed in real-time, which may not be acceptable in many in-vivo MRI scans. We present a much faster greedy algorithm, Parallel-OMP(P-OMP), which will solve the same problem in a few seconds. We also show that the accuracy of our approach is very similar to that of SOCP in both the single and multiple coil cases.

Theory

In parallel excitation pulse design with slice-selective subpulses, the amplitudes of the subpulses are computed by solving $\tilde{b} = \arg\min_b \|d - Ab\|_2$ where d is the desired in-plane excitation pattern, $A = [S_1F, S_2F, \ldots S_RF]$ is a stack of matrices where S_i is a diagonal matrix of the i-th coil's sensitivity pattern, and F is a 2D-Fourier encoding matrix restricted to the support of d. The vector $b = [b_1; b_2; \ldots; b_R]$ is a vertically concatenated vector where b_i is a vector composed of complex amplitudes of slice-selective pulses transmitted by the i-th coil and its element indices denote phase encoding locations. Each column of F represents a candidate phase-encoding location. Therefore, enforcing sparsity in phase encoding locations can be viewed as selecting a minimal number of columns of F such that a new matrix of selected columns, $\hat{F}$, can be used to form A in place of F to span d or its close approximation. Our algorithm is an iterative procedure where at each step, we choose columns of F that span a large portion of the residual of d, and then add those columns to $\hat{F}$, and update the residual by performing an orthogonal projection of d onto $\hat{A} = [S_1\hat{F}, \cdots, S_R\hat{F}]$.

P-OMP algorithm layout

Set $\hat{F}$ an empty matrix. Set the initial residual $r_0 = d$. Set $k = 1$.

Loop until the magnitude of the residual $\|r_{k-1}\|$ is sufficiently small. {

- Compute cumulative correlation[1] between columns in F and r_{k-1}
- Pick columns whose cumulative correlation is bigger than a threshold
- Add selected columns to $\hat{F}$ and do orthogonal projection of d onto $\hat{A} = [S_1\hat{F}, \cdots, S_R\hat{F}]$
- Set the new residual $r_k = d - \hat{A}\tilde{b}$. }

[1]Cumulative correlation : Let f_i be i-th column of F. Let j be a coil index

P-OMP(l_p) : $\sum_j \left|\langle S_j f_i, r_k \rangle\right|^p$

P-OMP(projection) : do orthogonal projection of d onto, $\{S_1 f_i, S_2 f_i, \ldots, S_R f_i\}$ and compute the magnitude of projected vector.

One important distinction that our problem makes from standard sparse approximation problems is that choosing one column of F corresponds to R columns in the approximation of d, because A has R columns associated with the selected column in F. Therefore, to assess how much of the residual is spanned by each candidate column in F, we experimented with three different approaches as in the algorithm layout on the left: P-OMP(l_1), P-OMP-(l_2),and P-OMP(projection). We call this value cumulative correlation, and large correlation implies that a large portion of the residual can be spanned by the corresponding R columns. At each iteration of our algorithm, we compute the cumulative correlation of the candidate columns of F and pick the columns of cumulative correlation value above a certain threshold. Here we set the threshold to be 99% of the maximum cumulative correlation at each iteration. By utilizing an efficient conjugate gradient solver for least squares calculations as well as implicitly constructing the matrix A using FFT's, we were able to ensure that our implementation is both fast and very memory efficient.

Experiments and Discussion

To compare the performance between the convex optimization method[3] and our method, we measured the Normalized Root Mean Squared Error(NRMSE) of the approximated pattern with these two methods as we increment the number of phase encoding locations. Fig1. and Fig.2 show the NRMSE measured in the computer simulation. On a computer with Intel Core2 Quad CPU 2.4GHz, 4GB RAM and Matlab 7, we ran the above two algorithms to excite a uniform circular pattern of a radius 10.125cm with a single coil and with 8 coils. The field of excitation is 24cm by 24cm over a 64x64 sampling grid. As seen in the Figs.1 and 2, the NRMSE curves along the number of chosen phase-encoding locations show that our method shows compatible accuracy to the convex optimization. Also the Table 1 shows that our method runs much faster than the convex optimization on the same problem instance.

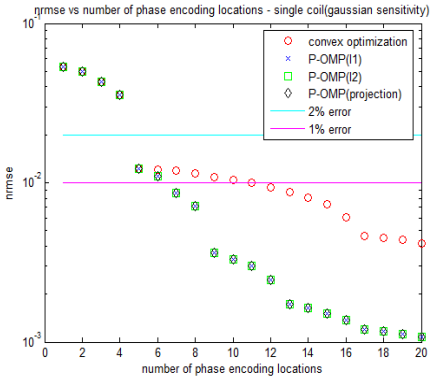

Fig 1. NRMSE in a single coil excitation

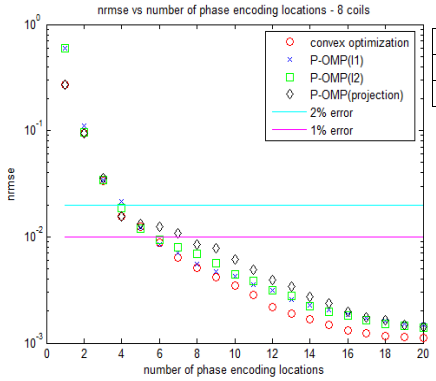

Fig 2. NRMSE in 8 coils, parallel excitation

	Convex optimization	OMP
Single coil	16 min	Less than 5sec
8 coils	48 min	Less than 25sec

Table 1. Runtime of OMP and Convex optimization method

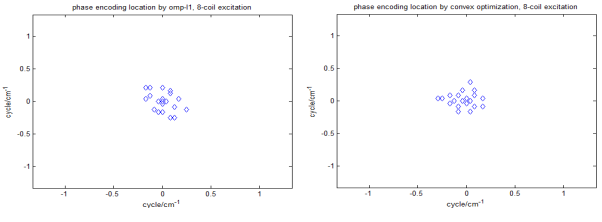

Fig 3.Phase encoding locations- 8 coils:P-OMP v.s. Convex optimization

Conclusion

Our P-OMP-based method to enforce sparse phase-encoding locations in parallel excitation performs comparably with the SOCP solver in terms of accuracy. However, our greedy approach is significantly faster, and thus, is more desirable in applications where on-line computation of the RF pulse is crucial.

References & Acknowledgements: [1] Tropp, *Signal Process.*,vol 86,no.3, pp.572-588,Apr.2006. [2] Zhang, *Mag. Res. Med.*,57(5):842-847,Apr.2007. [3]Zelinski, *IEEE Trans.Med.Imag.*, 27(9):1213-1229,Sept.2008

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