

# Reconstruction of NMR Spectra from Truncated Data with the Fast Padé Transform

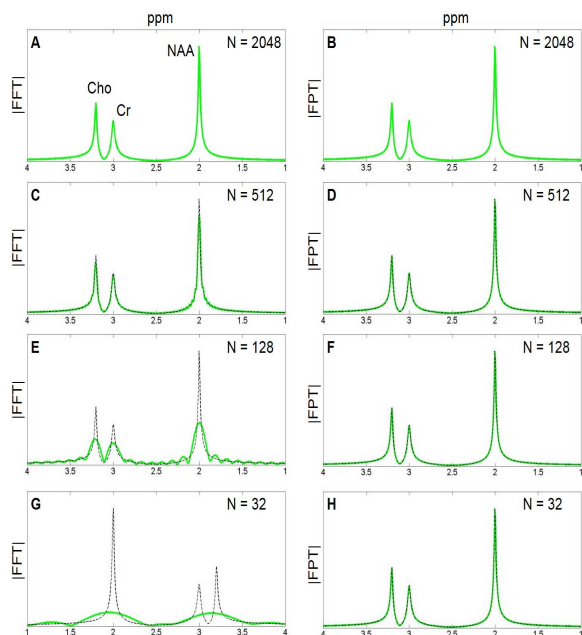
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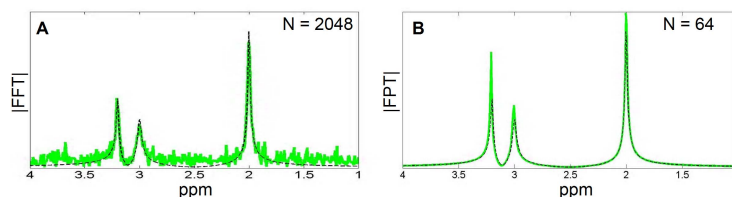
**Introduction:** MR spectroscopy can be used to identify various metabolites and their quantities *in vivo* and provide biochemical information that can be correlated with disease. For example, MR spectroscopy of the brain can show levels of creatinine (Cr), choline (Cho), and N-acetyl aspartate (NAA), which are correlated with the presence or absence of tumor. One of the major drawbacks in MR spectroscopy is low signal to noise ratio (SNR). To overcome this, the FID signal is acquired multiple times over a lengthy amount of time. In general, accuracy of MR spectroscopy must be traded off with long examination time. Current methods of spectroscopy rely on taking the fast Fourier transform (FFT) of the FID signal, which thus places fundamental constraints on the length of data acquisition to overcome low SNR. An alternative to performing the FFT is to use Padé approximants (PA) (1). Recently, Belkic defined an analytical continuation variant of the fast Padé transform (FPT) (2,3) as a possible alternative to the FFT for processing FID signals. We further investigate the properties of the FPT to determine its applicability in MR spectroscopy using shorter acquisition times.

**Methods:** The FPT was coded using MATLAB using a simulated FID signal as input. The FPT code uses singular value decomposition to solve the over-determined system of linear equations required to solve for the denominator expansion coefficients. The denominator coefficients were then used directly to solve for the numerator coefficients. The FID signal was artificially constructed as a sum of damped sinusoids containing metabolite signal data (T2 decay constants and frequencies) consistent with NAA, Cr, and Cho. The number of FID data points varied from  $N = 32 - 2048$ . Time resolution was 1 ms.  $K$ , the degree of the polynomials, was found using Hankel determinants (2). When corrupting simulated FID signals with random noise, SNR was chosen to be 10. Zero padding was used with truncated FIDs processed by the FFT. No zero padding whatsoever was done for FIDs processed by the FPT. A noise-free spectrum with  $N = 2048$  was obtained using the FFT and used as a gold standard to compare Fourier transform and FPT generated spectra from truncated FIDs.

**Results:** Figure 1 shows the effect of FID truncation on FFT and FPT processed spectra. Frequency spectra in green are constructed from a varying number of data points ( $N$ ) from the FID signal. In black is the FFT spectrum for  $N = 2048$ , the gold standard. Peaks representing NAA, Cr, and Cho are labeled. A, C, E, and G are spectra generated using the FFT. B, D, F, and H are spectra generated using the FPT. For decreasing  $N$ , the peaks in the FFT spectra become diminished and broaden. However, for decreasing  $N$  the FPT spectra retain their peaks and widths consistent with the gold standard. Figure 2 emphasizes the utility of the FPT to acquire many spectra with short acquisition time. Figure 2A in green shows a FFT spectrum from a FID with noise and  $N = 2048$ . Figure 2B in green shows a FPT spectrum from an average of 32 different FIDs with noise and with  $N = 64$ . In black is the gold standard spectrum. For both cases, the total acquisition time was equal. Using the same amount of acquisition time as the signal used to form the FFT spectrum, the FPT spectrum is surprisingly smooth compared to the FFT spectrum.



**Figure 1:** Comparison of FFT and FPT noise-free spectra for varying  $N$ .



**Figure 2:** Spectra from simulated FID signals corrupted with noise. FPT spectrum was formed from 32 acquisitions with  $N=64$ , while FFT spectrum was formed for a single FID of  $N=2048$ . Total acquisition time is the same.

**Discussion:** The FPT and FFT-processed spectra are similar for low amounts of truncation. However, peaks from spectra obtained using the FPT remain sharp for high amounts of truncation whereas spectra obtained using the FFT are limited by low resolution. Thus, there may be an advantage to using FPT for high amounts of truncation to shorten overall acquisition times in MR spectroscopy. As demonstrated in Figure 2, repeated sampling of a truncated FID and reconstruction with the FPT has significant noise advantage over conventional acquisition of a long FID and FFT reconstruction, with equal total acquisition time.

## References:

1. Baker, *Padé Approximants* 1996.
2. Belkic, *Nucl. Instr. And Meth. A.* 2004; 525:372-378.
3. Belkic, *Phys. Med. Biol.* 2006; 51:2633-2670.